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## **Credit-Implied Volatility**

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# Credit-Implied Volatility\*

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## Abstract

The pricing of corporate credit can be succinctly understood via the *credit-implied volatility* (CIV) surface. We invert it each month from the firm-by-maturity panel of CDS spreads via the Merton model, transforming CDS spreads into units of asset volatility. The CIV surface facilitates direct comparison of credit spreads at different “moneyness” (firm leverage) and time to maturity. We use this framework to organize the behavior of corporate credit markets into three stylized facts. First, CIV exhibits a steep moneyness smirk: Low leverage (out-of-the-money) CDS trade at a large implied volatility premium relative to highly levered (at-the-money) CDS, holding all other firm characteristics fixed. Second, the dynamics of credit spreads can be described with three clearly interpretable factors driving the entire CIV surface. Third, the cross section of CDS risk premia is fully explained by exposures to CIV surface shocks. Using a structural model for joint asset behavior of all firms, we show that the shape of the CIV surface is consistent with an aggregate asset growth process characterized by stochastic volatility and severe, time-varying downside tail risk. Lastly, we document these same CIV patterns among other credit instruments including corporate bonds and sovereign CDS.

**Keywords:** CDS, credit risk, implied volatility

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# 1 Introduction

We introduce the concept of a credit-implied volatility (CIV) surface. This framework organizes the behavior of corporate credit prices into a handful of easy-to-visualize facts. It also provides a clear visual diagnostic of where and why candidate credit risk models fail to match data.

Like its option-implied volatility analogue, CIV is inverted from the observable CDS spread to provide a relative measure of CDS value, after accounting for the effects of maturity, moneyness, and the risk-free rate via the [Merton \(1974\)](#) model. CIV is interpretable as a measure of asset volatility, answering the question: How much asset volatility does the Merton model require to justify a firm’s observed credit spread, given its leverage and contract maturity? The CIV surface provides a compact graphical summary for how the relative price of default insurance varies by time to maturity and “moneyness,” where moneyness is defined as the distance between current asset value and the default boundary. And as with options, CDS are available at a range of maturities for any single reference entity, thus CIV provides a direct means for comparing CDS value along the term structure. Yet CIV differs from option-implied volatility (OIV) in an important dimension. Options for a single underlying equity are available at a variety of strike prices, so the OIV curve (holding maturity fixed) also directly compares options with different moneyness for the *same* underlying. In contrast, a firm’s CDS contract is available at a single “strike.” The strike price for a credit instrument is the firm’s default boundary, which we proxy using firm leverage.<sup>1</sup> CDS trade at a wide range of maturities and on firms that differ greatly in their leverage. By jointly analyzing the entire universe of CDS, we trace out the CIV surface as a function relating CIV to firm leverage (moneyness) and to CDS maturity. And, because the “strike” price of a CDS is far deeper out-of-the-money than a firm’s equity puts, CIV gives insight into regions of the risk-neutral asset distribution that are impossible to infer from options prices.

Three main stylized facts emerge from our analysis and are organized around an understanding of the empirical CIV surface.<sup>2</sup> First, we identify a CIV moneyness “smirk.” That is, CIV is steeply decreasing in firm leverage. This may be surprising at first glance. After all, CDS *spreads* are naturally increasing in leverage—spreads of very low leverage firms are nearly zero and rise

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<sup>1</sup>Merton model spreads are homogenous of degree one in total assets and face value of debt. Spreads can therefore be modeled by normalizing total assets to unity and similarly normalizing the contract strike price (the face value of debt) by total assets. The resulting normalized strike price is the leverage ratio.

<sup>2</sup>This mirrors the option literature’s orientation around empirical regularities in option-implied volatilities.

monotonically with leverage, as leverage directly increases probability of default all else equal. But the downward sloping CIV smirk tells us that firms with low leverage in fact have unusually *high* CDS spreads in relative terms. That is, disproportionately high asset volatility is necessary to generate a high enough probability of default, and thereby justify the observed spread, given how little debt these firms carry. The CIV moneyness smirk is most steeply negative for CDS with one year to maturity, and gradually flattens out over longer maturities up to ten years. Said another way, the term structure of CIV is steeply negative for out-of-the-money (low leverage) credits, and is slightly upward sloping for at-the-money (high leverage) firms.

The above facts are about the unconditional average CIV surface. Because the surface is extracted from price data, it can be computed on any individual day. Our second stylized fact uses time variation in the surface to succinctly summarize the dynamics of CDS prices. CIV across all firms and maturities demonstrates large common (and persistent) fluctuations over time. In recessions, the overall level of the surface rises as default risk increases. At the same time, the moneyness smirk flattens, as relative insurance prices for highly levered credits rise faster than those of low leverage firms. This flattening is more pronounced for long maturity CDS, introducing variation in the slope of the CIV term structure, which becomes more negative in downturns. Fluctuations in moneyness and term structure slopes twist and untwist the surface over time. This twisting behavior indicates that more than one factor is necessary to describe CIV surface dynamics. Principal components analysis shows that three factors provide a nearly perfect description of the surface ( $R^2 = 99\%$ ). The estimated factors possess clear interpretations; the first PC lines up with the CIV surface level, the second PC with the term structure slope, and the third PC with the moneyness slope.

Our third stylized fact moves from studying credit price levels to analyzing credit risk premia. CDS Sharpe ratios line up monotonically in the leverage and maturity dimensions. Holding maturity fixed, Sharpe ratios are highest for low leverage CDS and gradually decline as leverage rises. Holding moneyness fixed, they are highest for one-year CDS and decline with maturity. The CIV surface plays a fundamental role in understanding this pattern of risk compensation. Differences in average CDS returns are nearly perfectly explained by heterogeneity in their exposures to the three common factors in the CIV surface. We show how the credit risk premium can be divided into two components. One is a standard premium arising from the CDS's exposure to overall asset growth

in the economy. The other is a variance risk premium earned from the embedded put option that makes CDS a natural hedge against volatility shocks. Empirically we find that the majority of the credit risk premium is compensation for bearing volatility risk.

What gives rise to the CIV moneyiness smirk? There are two natural candidate explanations. The first is firm heterogeneity—firms with different leverage differ on other dimensions as well. Most importantly, they may have riskier assets, which would be reflected in higher CIV. While there is some evidence that low leverage firms are riskier, this is not the primary explanation. We find that the CIV smirk is qualitatively unchanged after controlling for other firm observables, such as size, asset volatility, credit rating, and industry.

If firm heterogeneity is not the main driver of the CIV smirk, the most likely alternative is that firm-level asset growth is heavy-tailed. It is well known from the options literature that an implied volatility smirk (where implied volatilities are calculated under the Merton model’s normality assumption) arises when the true distribution is left-skewed and/or leptokurtic. This fact, coupled with the fact that CIV of all firms move in unison (via the three factors mentioned above), raises the intriguing possibility that a few aggregate state variables are simultaneously driving the asset growth distributions of all individual firms in the economy.

We explore this possibility in a structural model for the joint behavior of individual firms. We propose a parsimonious specification with the following ingredients. First, we include an aggregate asset growth process that is subject to stochastic volatility and jumps. Having at least two state variables that mean revert at different speeds (such as one volatility process and one jump intensity process) allows our model to match the term structure dynamics of CDS. Furthermore, jumps incorporate severe skewness and kurtosis needed to match the steepness of the moneyiness smirk. Next, to match the tight comovement of firms’ CDS spreads, asset growth processes for individual firms are linked together in two ways. They all have exposure to aggregate growth (as in the CAPM) so that firms inherit the higher moment dynamics of the aggregate asset. We also allow idiosyncratic asset growth to have higher moment dynamics that are correlated with the aggregate growth distribution, a feature strongly suggested by the data.<sup>3</sup> From here, we derive CDS spreads in the model using the original [Merton \(1974\)](#) assumption that debt is a put option on the firm’s

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<sup>3</sup>[Kelly and Jiang \(2014\)](#); [Kelly, Lustig and Van Nieuwerburgh \(2013\)](#); [Herskovic et al. \(2015\)](#); [Connor, Korajczyk and Linton \(2006\)](#) show that firm-level idiosyncratic volatilities and tail risks strongly correlate with aggregate market risks.

assets, where the strike price equals the firm’s debt value and default occurs only at maturity. This intentionally oversimplified contract structure provides a close quantitative match of the empirical CIV surface. In particular, cross-sectional heterogeneity and time series dynamics in credit spreads are driven only by firm leverage, CDS maturity, and exposure to a few aggregate state variables.

Our estimates point to the jump process as the single most important model ingredient for matching credit spreads. Furthermore, estimated jump parameters amount to a rare disaster specification. Aggregate jumps arrive on average once every 116 years and when they arrive they are cataclysmic, with an average log jump size of  $-71\%$ . This is perhaps unsurprising given the basic shape of the CIV moneyness smirk. Effective “strike” prices for CDS are much further out-of-the-money than the deepest out-of-the-money equity put. In order for low leverage firms to experience a default, nearly their entire asset value would need to be wiped out. Yet we see that the CIV smirk steepens in the low end of the leverage support, illustrating substantial mass in this left tail of the risk-neutral asset growth distribution. The estimated model formalizes the intuition that large crash risk is necessary to match the high CIV values observed for firms with leverage below 20%.

In an extension, we study CIV extracted from corporate bonds rather than CDS. Our main analysis focuses on CDS because they are standardized contracts, have fixed maturity structure, avoid issues of optionality/callability, and tend to be less subject to illiquidity concerns. However, CIV from corporate bonds share nearly identical patterns as those in CDS, so none of our documented patterns are isolated to the CDS market. In a second extension, we construct and analyze the sovereign CIV surface from a panel of 24 countries. Sovereign CIV also demonstrates the same qualitative patterns as corporate CIV.

## 1.1 Contributions and Literature Review

Our paper is related to several lines of research in the empirical credit risk literature. At the broadest level, our work continues in the [Merton \(1974\)](#) tradition of using option pricing machinery for understanding the behavior of credit claims. Previous work estimates asset volatility from bonds to quantify default risk (e.g., [Vassalou and Xing, 2004](#)). We offer the new insight that a complete surface of credit-implied volatility may be extracted from CDS on diverse firms and across maturities. This allows a credit analyst to plainly visualize and interpret the drivers of default risk

that join together credit price behavior throughout the economy.

Next, our statistical factor decomposition of the CIV surface shows that surface dynamics are fully captured with a small number of common factors.<sup>4</sup> Similar decompositions of the S&P 500 option-implied volatility surface have been studied by [Cont, Da Fonseca et al. \(2002\)](#) and [Andersen, Fusari and Todorov \(2015\)](#). [Andersen, Fusari and Todorov \(2015\)](#) use statistical OIV factors to draw insights into option price dynamics and use this to develop a significantly improved structural specification. In the same vein, our statistical analysis of CIV patterns lends new perspective on the structural drivers of corporate credit risk. For a model to describe the joint behavior of CDS for all firms and maturities, it requires at a minimum the following features. First, because CIVs of individual firms demonstrate minimal idiosyncratic variation, firms’ asset distributions must be primarily driven by aggregate state variables.<sup>5</sup> Second, the asset growth process must incorporate heavy-tailed shocks in order to generate a moneyness smirk. We rely on stochastic volatility and jumps in aggregate asset growth to achieve this.<sup>6</sup>

Non-Gaussian models have proven useful for understanding spreads because pure Gaussian risk is incapable of generating large enough credit spreads at short maturities and for highly rated firms—this is the so-called “credit spread puzzle” of [Jones, Mason and Rosenfeld \(1984\)](#) and [Huang and Huang \(2012\)](#). The traditional puzzle can be equivalently restated as short maturity and highly rated debt have abnormally high CIV. Our CIV surface, however, offers a more complete perspective on the credit spread puzzle by providing a global map of relative credit prices. Steep differences in relative credit prices do not appear just at very short maturities or just for AA credits, but are prevalent throughout the maturity-moneyness plane. We also refine the traditional facts by showing that leverage, as opposed to rating or other notions of credit quality, most accurately

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<sup>4</sup>[Collin-Dufresne, Goldstein and Martin \(2001\)](#) document a factor structure in residuals from a regression of bond spreads onto theoretically motivated determinants of spreads, but find this factor unrelated to commonly studied macro and financial factors. In contrast, the factors we document possess clear economic meaning associated with overall credit conditions and differences in CIV across maturity and moneyness.

<sup>5</sup>Previous credit risk models along these lines include [Coval, Jurek and Stafford \(2009\)](#) and [Cremers, Driessen and Maenhout \(2008\)](#) who jointly model aggregate and idiosyncratic jumps in asset value. [Ericsson et al. \(2009\)](#) propose a related reduced form model with distinct aggregate and idiosyncratic default intensity processes. [Christoffersen, Fournier and Jacobs \(2013\)](#) and [Kelly, Lustig and Van Nieuwerburgh \(2016\)](#) study related models for pricing the cross section of options.

<sup>6</sup>Several papers have investigated the usefulness of standard option pricing models for modeling credit risk. Examples of structural models with jumps include [Mason and Bhattacharya \(1981\)](#); [Zhou \(2001\)](#); [Delianedis and Geske \(2001\)](#); [Hilberink and Rogers \(2002\)](#); [Cremers, Driessen and Maenhout \(2008\)](#); [Chen and Kou \(2009\)](#); [Huang and Huang \(2012\)](#). [Huang \(2005\)](#) proposes a model with stochastic asset growth volatility and [Huang and Zhou \(2008\)](#) argue that empirical evidence supports a role for stochastic asset volatility, though they do not model this.

captures differences in relative credit costs across firms. Perhaps our most novel enrichment of traditional credit spread puzzle facts is understanding state dependence in relative credit prices. We show that at least two state variables are needed to match persistent fluctuations in the level and curvature of the CIV surface, and we show that these factors are interpretable as time-varying volatility and time-varying crash risk in aggregate asset growth.<sup>7</sup>

Lastly and most broadly, we contribute to a large literature examining the empirical determinants of credit spreads. Most of the literature takes one of two approaches to empirical analysis. The first calibrates structural models to “representative” credit securities in each rating category, seeking to match, for example, unconditional average credit spreads for a given rating, rather than matching spreads firm-by-firm and month-by-month. A benefit of the calibration approach is that models can be evaluated based on their ability to match observed spreads while respecting historical default rates. Examples of this approach include [Cremers et al. \(2008\)](#); [Chen, Collin-Dufresne and Goldstein \(2009\)](#); [Huang and Huang \(2012\)](#). A shortcoming is that credit risk dynamics cannot be directly analyzed with this approach due to the inherent infrequency of historical defaults.

Our approach differs in that we target CDS spreads of all firms, all maturities, and in *every* time period.<sup>8</sup> Despite the strenuous demands this approach puts on our model, we show that restrictions implied by our model appear well satisfied in the data. Rather than separately estimating the physical asset growth distribution and risk premia, we directly estimate the risk-neutral asset growth distribution, and then compare estimated risk-neutral moments to physical moments *ex post*. In particular, we investigate risk premia by studying average realized CDS returns, and leave a detailed investigation of model-based risk premia to future work.<sup>9</sup>

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<sup>7</sup>Our focus on a small number of common aggregate risk factors and the model’s prediction of economy-wide high credit spreads during risky episodes like the financial crisis capture the countercyclical spread patterns emphasized in macroeconomic models of credit risk such as [Chen, Collin-Dufresne and Goldstein \(2009\)](#), [Chen \(2010\)](#), and [Bhamra, Kuehn and Strebulaev \(2010\)](#). Our paper is also related to [Chen et al. \(2015\)](#) who study the interaction of default and liquidity risks over the business cycle, to [Seo and Wachter \(2015\)](#) who study the value of CDX tranches through the lens of a rare disaster endowment economy, and to [Engle and Siritwardane \(2015\)](#) who study the interaction between volatility and leverage in a Merton model.

<sup>8</sup>[Huang and Zhou \(2008\)](#) and [Feldhütter and Schaefer \(2014\)](#) also target match the cross section and time series behavior of credit spreads. Neither paper models the joint behavior of firms in the panel. [Feldhütter and Schaefer \(2014\)](#) argue that individual firm calibration in large part resolves the credit spread puzzle, and resurrects the Merton model as a viable descriptor of credit spreads. While we take a similarly disaggregated approach, the stark money skew and CIV dynamics that we find immediately illustrate failures of the Merton model in the same manner that option-implied volatility patterns reject the Black-Scholes model.

<sup>9</sup>This approach is motivated in part by the great difficulty of accurately estimating physical jumps and stochastic volatility from realized asset growth data, which is observed infrequently and with error (due to reliance, for example, on quarterly balance sheet information). Comparing risk-neutral estimates to coarse measures of physical moments confirms that the risk-neutral model does not unreasonably deviate from physical data.

Our model produces a highly accurate match of the CIV surface evolution, despite abstracting from a number of credit market features studied in previous work.<sup>10</sup> Our model’s close fit of spread data, despite leaving aside more subtle nuances of credit markets and credit instruments, is consistent with the findings of [Culp, Nozawa and Veronesi \(2014\)](#).<sup>11</sup> In robustness tests we study how CIV is affected by mean reversion in leverage, as emphasized by [Collin-Dufresne and Goldstein \(2001\)](#), and find that this cannot account for the shape of the CIV surface.

Our work is also related to the second common approach in empirical analysis of spreads, which abandons the structural setting for linear panel regression. This method has the benefit of targeting the full cross section and time series of spreads and its flexibility has proven useful for uncovering problematic assumptions of structural models.<sup>12</sup> In this area, our paper is most closely related to [Ericsson, Jacobs and Oviedo \(2009\)](#) who emphasize the role of traditional theoretical determinants of default risk, especially volatility and leverage. These studies, which most frequently regress spreads onto firm-level characteristics, differ from our emphasis on common aggregate determinants of firm-level credit spreads. Perhaps most interestingly, we show that our structural model achieves explanatory power competitive with regression specifications while maintaining the rigor and parsimony of structural model restrictions.

In the remainder of the paper, we first present our method for constructing CIV (Section 2) and describe the data we use for our analysis (Section 3). Next, we analyze the empirical properties of the CIV surface and credit risk premia (Section 4). We specify, estimate, and report results for our cross-sectional CDS pricing model in Section 5. Finally, we analyze extensions to bond markets and sovereign CDS markets in Section 6, and analyze the behavior of CIV in sector and credit rating

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<sup>10</sup>Examples of such features include stochastic interest rates (e.g., [Shimko and Tejima \(1993\)](#)), endogenous default (e.g., [Leland \(2006\)](#)), debt covenants (e.g., [Black and Cox \(1976\)](#)), taxes (e.g., [Elton et al. \(2001\)](#)), coupon structure (e.g., [Bielecki and Rutkowski \(2002\)](#)), maturity structure (e.g., Moody’s KMV), recovery rates (e.g., [Altman, Resti and Sironi \(2004\)](#)), and others.

<sup>11</sup>These authors show that “pseudo-bonds,” constructed from equity options and risk free debt to mimic corporate credit, can match many empirical regularities in corporate credit markets despite their lack of direct credit risk exposure, suggesting that first order behavior of debt prices is unlikely to be driven by contractual or trading environment features that are unique to corporate credit. Other work examining the link between a firm’s option and credit prices includes [Hull, Nelken and White \(2004b\)](#); [Cremers et al. \(2008\)](#); [Carr and Wu \(2011\)](#); [Cao, Yu and Zhong \(2010\)](#); [Wang, Zhou and Zhou \(2013\)](#).

<sup>12</sup>Examples of this approach include [Collin-Dufresne and Goldstein \(2001\)](#) who emphasize local market imbalances as drivers of spreads, [Elton et al. \(2001\)](#) who document the role of taxes and risk premia, [Campbell and Taksler \(2003\)](#) who illustrate the role of idiosyncratic volatility, [Longstaff, Mithal and Neis \(2005\)](#) who study the role of liquidity, [Zhang, Zhou and Zhu \(2009\)](#) who document the role of realized volatility and jump risks, [Cremers et al. \(2008\)](#) who illustrate the role of option-implied volatility and jump risks, and [Siriwardane \(2015\)](#) who documents the role of financial intermediary capital. [Doshi et al. \(2013\)](#) propose a hybrid approach, embedding typical regression covariates in a no-arbitrage credit pricing model.

subsamples.

## 2 Credit Implied Volatility

In this section we describe our construction of credit-implied volatility (CIV) and document its behavior in the data. CIV measures the asset volatility implied from CDS spreads via the Merton model formula for credit spreads. By starting from this simple baseline model, we can use the model’s implied volatilities to visualize the relative expensiveness of insurance against credit defaults, after accounting for the effects of maturity, moneyness, and the risk-free rate. Our CIV construction is analogous to that of Black-Scholes implied volatilities for options, with modifications for the credit setting and the interim swap payments of CDS contracts.

### 2.1 Definition of Credit Implied Volatility

In the seminal [Merton \(1974\)](#) theoretical framework, firm debt is equivalent to a portfolio of a risk free bond and a short put on the firm’s assets. Debt takes an especially simple form. At maturity  $T$ , debtholders receive either the full face value,  $D$ , or the remaining asset value  $A_T$  if the firm defaults (i.e., if  $A_T < D$ ).

CIV is derived from the valuation formula for the put component of the firm’s debt. As the name suggests, a CDS contract is not structured as a bond but as a swap with intermittent spread payments. We use a common approximation that equates the value of the put option implicit in a firm’s debt to the capitalized value of future CDS spread payments following, for example, [Hull, Nelken and White \(2004a\)](#) and [Friewald, Wagner and Zechner \(2014\)](#). In particular, CDS spreads in the Merton model are given by the function

$$s(\sigma_A, L, T - t, r) = -\frac{1}{T - t} \ln \left( N(d_2) + \frac{N(-d_1)}{L} \right) \quad (1)$$

where the firm’s leverage is defined as  $L = A/De^{-r(T-t)}$ , its instantaneous asset volatility is  $\sigma_A$ ,  $r$  is the interest rate, and  $N(\cdot)$  is the cumulative normal distribution with

$$d_1 = \frac{-\ln L}{\sigma_A \sqrt{T - t}} + \frac{1}{2} \sigma_A \sqrt{T - t}, \quad d_2 = d_1 - \sigma_A \sqrt{T - t}.$$

Formula (1) adapts the Merton bond price formula to the swap structure of CDS. The spread is monotonically increasing in the volatility of underlying assets,  $\sigma_A$ , so  $\sigma_A$  can be numerically inverted from the observed CDS spread as long as  $T - t$ ,  $r$ , and  $L$  are known. We define this inverse as credit-implied volatility,  $\text{CIV}(\tilde{s}, L, T - t, r) = s^{-1}(\tilde{s}, L, T - t, r)$ , where  $\tilde{s}$  is the observed credit spread.

### 3 Data

We collect month-end CDS spreads and credit ratings from the Markit database from January 2002 to December 2014. We study CDS with maturities of 1, 3, 5, 7, and 10 years.<sup>13</sup> We match CDS spreads to CRSP/Compustat monthly equity market data and quarterly accounting data by the reference entity’s cusip, and we lag accounting data by 3 months. We apply standard filters from the literature that limit the influence of contracts that suffer from poor liquidity or are in financial distress.<sup>14</sup> We construct monthly observations based on the last available daily data each month for each company. The final unbalanced panel consists of a 156 month time series and a cross-section of 530 companies.<sup>15</sup>

The two central CDS characteristics in our analysis are firm leverage and CDS contract maturity. We define leverage as

$$L = \frac{\text{Book debt}}{\text{Market equity} + \text{Book debt}}$$

where book debt value is the sum of short-term and long-term debt in Compustat (variables `dlcq` and `dlttq`), and market equity value is shares outstanding times price per share in CRSP.<sup>16</sup> Time to

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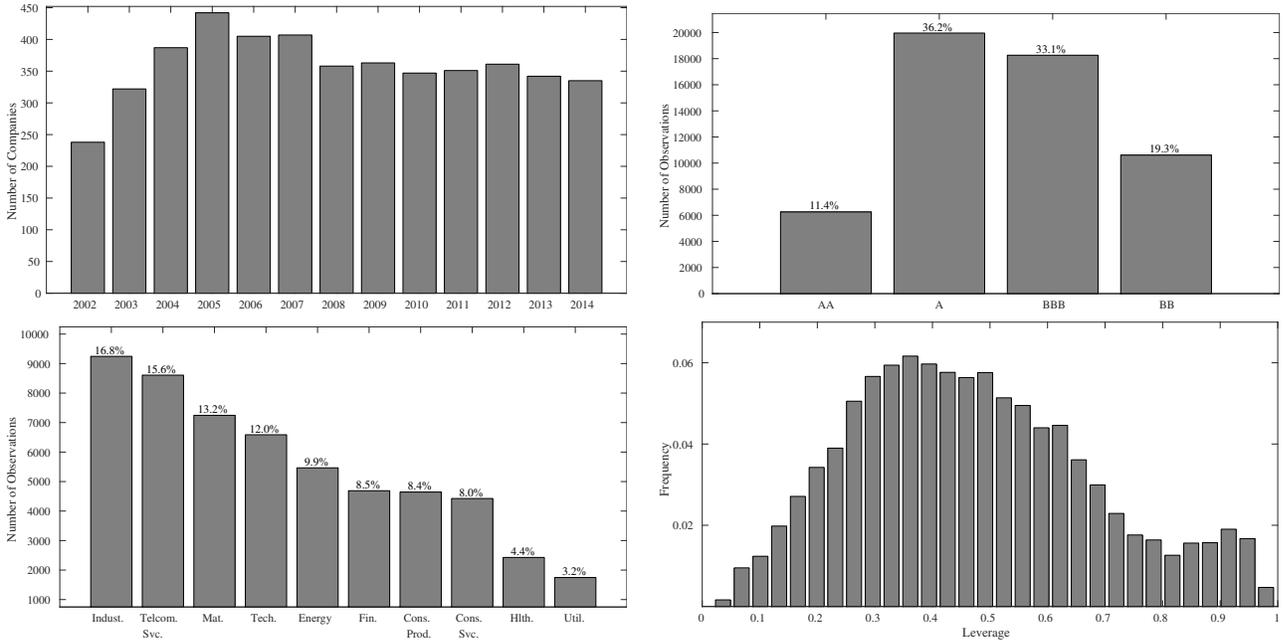
<sup>13</sup>We use contracts with the most common default definition at each point in time. This is the MR clause from January 1, 2002 to March 15, 2009, the XR clause from March 15, 2009 to September 22, 2014, and the XR14 clause from September 23, 2014 onward.

<sup>14</sup>We exclude the two lowest ratings bins in our data (B and CCC), spread observations reported by fewer than three dealers, spreads greater than 10%, and firms with fewer than 12 months of non-missing spreads. This eliminates approximately two percent of our observations.

<sup>15</sup>Much of the previous work on credit risk studies bond data. We study bond-based CIV in Section 6 and find very similar patterns to those for CDS. We prefer to study CDS when constructing CIV because CDS are standardized contracts with essentially constant maturity and no call premiums. Also, the credit spread calculated from a bond price depends on the bond’s liquidity and involves an assumption about the benchmark risk-free rate, and these issues are more or less avoided when studying CDS.

<sup>16</sup>Unlike equity options in which the strike price is defined in the contract, the strike price in the Merton model and in the model of Section 5 is less well defined. Our chosen definition of the default boundary is the level of total balance sheet debt, but robustness analysis suggests that our findings are not sensitive to this choice. For example, instead of using total debt, we obtained similar quantitative results when we instead use total liabilities as the default boundary. We also find that the CIV smirk is similar within industry. To the extent that mismeasurement in moneyness has an industry element (e.g. off-balance-sheet debt tends to be prominent among financials and operating leases among

Figure 1: SUMMARY STATISTICS



**Note.** Statistics for merged sample of 530 firms from 2002 to 2014.

maturity is an observable contract feature. Risk-free rate data are from the monthly term structure of constant maturity Treasury bonds in the H.15 data set of the Federal Reserve Board.<sup>17</sup> With these inputs, we numerically invert CIV from observed spreads via the Merton model CDS spread formula in (1).

Figure 1 plots summary statistics for our sample. The number of firms available each year is approximately 350, ranging from just over 250 in 2002 to nearly 450 in 2005. Credit ratings range from AA to BB, with most observations coming from intermediate credits (69.3% for A and BBB firms). The lower left panel shows the distribution of observations across industries, and the lower right panel shows the distribution by leverage ratio. Few firms have leverage below 10% or above 90%.

consumer non-durables), our industry-level analysis allays some fears that debt mismeasurement drives the smirk.

Vassalou and Xing (2004) provide an alternative method to estimate firm debt (along with firm asset volatility) using firm equity and equity volatility. We choose to instead estimate leverage from firms' balance sheets in the interest of transparency and ease of implementation.

<sup>17</sup>The risk-free discount function can be extracted also from Libor and interest rate swaps. However, Duffie (1999) shows that the risk-free discount function plays a negligible role in pricing credit default swap spreads. Therefore, the choice of measurement has little impact on pricing errors.

## 4 The Credit Implied Volatility Surface

The credit-implied volatility surface is a function that maps a CDS spread into asset volatility units given the underlying firm’s leverage, the CDS contract maturity, and assumptions of the Merton model. We first describe the unconditional properties of the surface and then document its dynamic behavior.

### 4.1 Unconditional Properties of CIV

Two basic features summarize the unconditional average shape of the CIV surface. The first we refer to as the CIV *moneyiness smirk*. It describes average CIV as a function of the leverage of the reference firm, holding CDS maturity fixed. The standard notion of moneyiness in the option literature is the ratio of the strike price to the underlying price. In the Merton model, default occurs when assets fall below the face value of its debt, so the firm’s leverage ratio describes “moneyiness” of the put option implicit in its debt. For highly levered firms, the ratio of debt to assets is close to one so there is little buffer between asset value and the default boundary. In this case, a CDS is nearly “at-the-money” (ATM). On the other hand, a firm with leverage near zero would need to suffer catastrophic deterioration of its assets before reaching default, thus its debt is “out-of-the-money” (OTM).

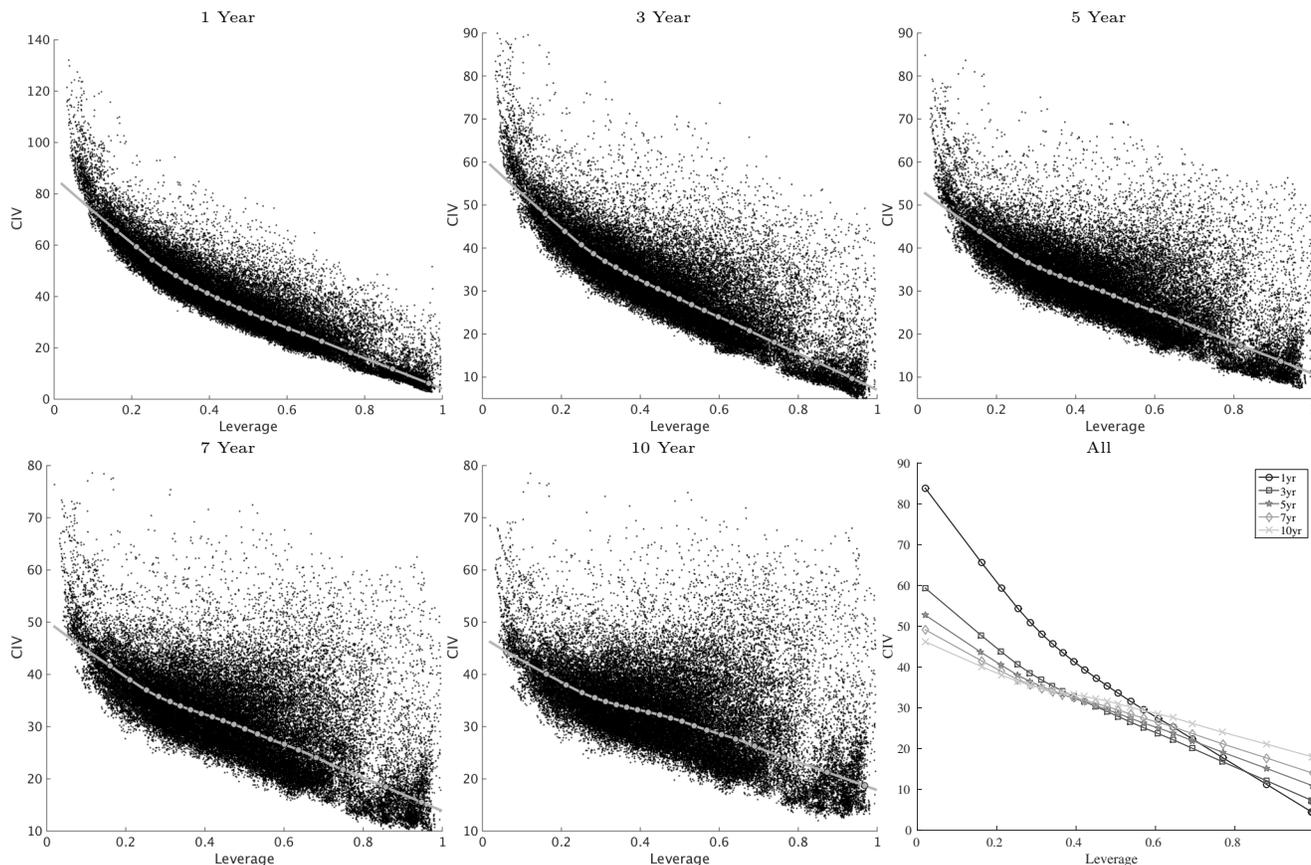
Figure 2 shows scattergrams of CIV versus leverage pooling all firm-month observations and broken out by CDS contract maturity. In each panel we fit a non-parametric curve to the CIV-leverage relationship.<sup>18</sup> Asset volatilities implied from CDS possess a clear smirk pattern with respect to moneyiness. The figures shows that, from the point of view of the Merton model, one needs to assume disproportionately high values of asset volatility in order to match the CDS prices for firms with low leverage. The cost of default insurance implicit in CDS spreads is thus relatively more expensive for deeper OTM credits. This is the credit risk analogue of the implied volatility smirk in equity options, which says that the Black-Scholes model requires relatively high values of equity return volatility in order to match the prices of deep out-of-the-money puts, all else equal.

The second feature of the surface is the CIV *term structure*, which describes the average CIV

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<sup>18</sup>The non-parametric curve represents the local average value of CIV at each value of leverage, and is calculated with Matlab’s `smooth.m` function using the robust weighted least squares (“rlowess”) option and span parameter of 0.5.

Figure 2: CREDIT IMPLIED VOLATILITY SMIRK



**Note.** Pooled scatter plots of monthly CIV versus firm leverage. The gray line is a fitted non-parametric curve. The lower right panel overlays the fitted CIV curve at all maturities to trace out the CIV surface.

pattern across maturities, holding leverage fixed. The lower right panel of Figure 2 summarizes the behavior of CIV in the term structure dimension. It overlays the fitted CIV smirk for each maturity. The smirk is steepest for one-year CDS, and monotonically flattens as the contract maturity lengthens. On average, the CIV term structure is therefore steeply downward sloping for the deepest OTM (low leverage) CDS, and is slightly upward sloping for ATM (high leverage) CDS.

The CIV surface is a more complicated object than the option-implied volatility surface. Options contracts at multiple maturities and strike prices are available for any single underlying firm, allowing one to trace out the volatility surface holding the underlying firm fixed. For CDS, however, there are multiple maturities but only one “strike price” per firm. The scatter plots in Figure 2 require deeper investigation because they pool together observations for all firms, and therefore mix

many dimensions of heterogeneity such as industry, credit rating, and fundamental riskiness. They are also heterogeneous in the time dimension, pooling observations over all periods so that data points from the financial crisis are not differentiated from observations during the great moderation.

We find that the basic shape of the CIV surface is largely unchanged after exhaustively controlling for non-leverage dimensions of CDS heterogeneity. To establish this, we construct an adjusted version of CIV that controls for heterogeneity across observations *except* firm leverage. We regress CIV onto firm-level characteristics that include credit rating (six indicator variables), sector (ten indicator variables), size (log total assets), and asset riskiness (monthly realized volatility, beta, skewness, kurtosis of firms' total asset returns). Appendix A describes our construction of daily realized asset returns that we use to calculate asset riskiness. We include month fixed effects to absorb time variation in CIV that is common across firms (due to, for example, fluctuations in macro volatility). Finally, we control for maturity effects by running separate regressions for each maturity. These regressions take the form

$$\text{CIV}_{i,\tau,t} = \delta_{0,\tau} + \delta'_{1,\tau} [\text{Size, Beta, Vol, Skew, Kurt}]'_{i,t} + \text{Rating FE} + \text{Sector FE} + \text{Month FE} + \epsilon_{i,\tau,t}, \quad (2)$$

where subscript  $i$  denotes the firm,  $\tau$  denotes the CDS contract maturity, and  $t$  denotes the month of an observation. We define the final characteristic-adjusted CIV measure as the regression residual,

$$\widehat{\text{CIV}}_{i,\tau,t} = \epsilon_{i,\tau,t}.$$

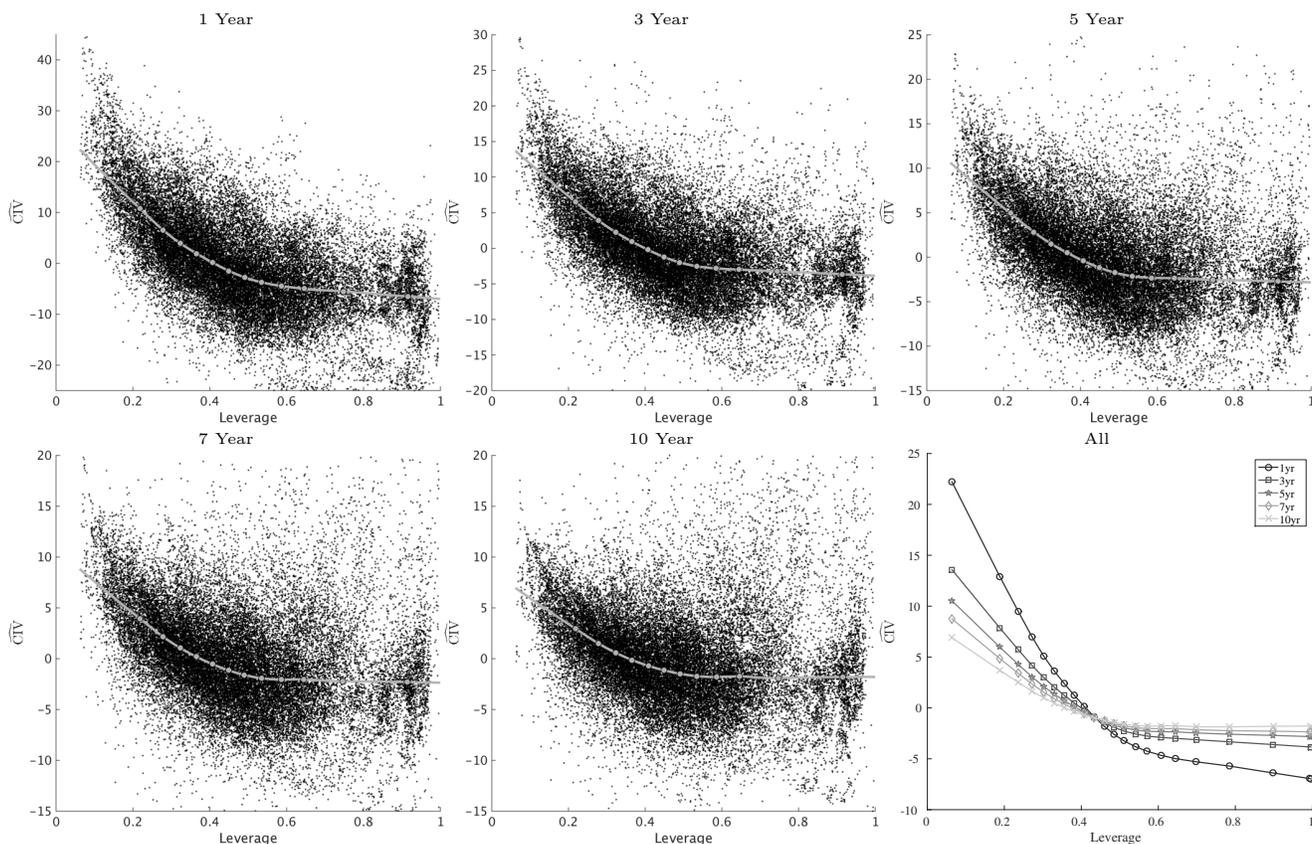
$\widehat{\text{CIV}}$  isolates the behavior of CIV that is uniquely associated with firm leverage, stripping out the influence of other observables.<sup>19</sup>

Figure 3 shows scatter plots of  $\widehat{\text{CIV}}$  versus firm leverage. The  $\widehat{\text{CIV}}$  scatter possesses the same basic smirk shape as the raw CIV scatter in Figure 2, indicating that the comparatively high price for default insurance of low leverage firms does not simply reflect differences in the size, industry, equity risk, or credit ratings of low leverage firms. Nor does it appear that the smirk arises from mixing observations at different points in the business or credit cycle, based on our inclusion of month fixed effects.<sup>20</sup>

<sup>19</sup>We present detailed results from these regressions, including coefficients and  $R^2$ , in Table 2.

<sup>20</sup>We have also constructed  $\widehat{\text{CIV}}$  controlling for measures of realized equity riskiness, rather than asset riskiness. In

Figure 3: CREDIT IMPLIED VOLATILITY SMIRK (HETEROGENEITY-ADJUSTED)



**Note.** Pooled scatter plots of monthly  $\widehat{CIV}$  versus firm leverage. The gray line is a fitted non-parametric curve. The lower right panel overlays the fitted  $\widehat{CIV}$  curve at all maturities to trace out the  $\widehat{CIV}$  surface.

Table 1 compares the shapes of the CIV and  $\widehat{CIV}$  smirks by reporting differences in implied asset volatility at different values of leverage. The first row shows the difference in CIV for firms with 10% leverage and firms with 90% leverage—this describes the overall magnitude of the smirk slope. The raw CIV smirk is two to three times as steep as the that for  $\widehat{CIV}$ . This row indicates that at least some portion of the raw CIV moneyness slope is attributable to other dimensions of firm heterogeneity that correlate with leverage.<sup>21</sup>

A closer look shows that the primary difference between the CIV and  $\widehat{CIV}$  in Figures 2 and 3 comes from the fact that the  $\widehat{CIV}$  smirk has more curvature, as shown in the second and third rows general, equity risk has slightly less explanatory power for CIV than asset risk, as shown in the regressions of Table 2. We opt for the more conservative  $\widehat{CIV}$  construction using asset risk measures.

<sup>21</sup>This is consistent, for example, with Choi and Richardson (forthcoming), who show that high leverage firms have lower average asset volatility, and with Schwert and Strebulaev (2014), who show that high leverage firms have lower asset betas.

Table 1: MONEYNES SLOPE

Leverage	1 Year		3 Year		5 Year		7 Year		10 Year	
	CIV	$\widehat{\text{CIV}}$	CIV	$\widehat{\text{CIV}}$	CIV	$\widehat{\text{CIV}}$	CIV	$\widehat{\text{CIV}}$	CIV	$\widehat{\text{CIV}}$
10% – 90%	63.3	25.9	41.4	15.5	33.0	11.9	27.7	9.9	22.1	7.8
10% – 50%	39.4	22.4	24.5	14.0	18.6	11.0	15.1	9.3	11.5	7.5
50% – 90%	23.9	3.4	16.9	1.5	14.4	0.8	12.6	0.6	10.6	0.3

**Note.** Comparison of the slope of fitted non-parametric smirk curves for CIV and  $\widehat{\text{CIV}}$ . The first row reports the difference in the fitted value of average CIV for 10% leverage firms versus that for 90% leverage firms, and likewise for  $\widehat{\text{CIV}}$ . The second and third rows describe curvature of the smirk by separately calculating its slope for low leverage firms (10% – 50%) and high leverage firms (50% – 90%). Slopes are separately reported for each maturity.

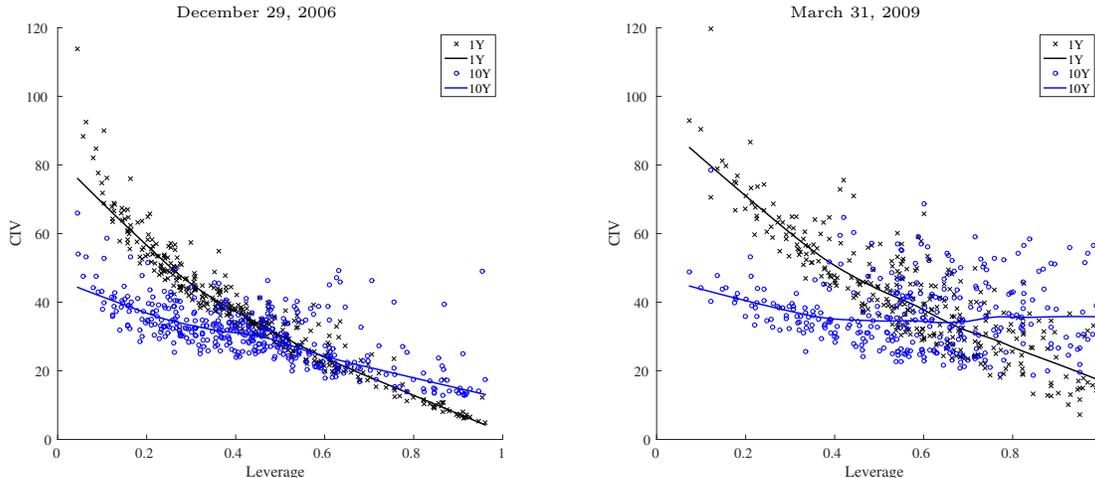
of Table 1. The  $\widehat{\text{CIV}}$  smirk slope is almost entirely concentrated among low leverage firms (between 10% and 50% leverage), and looks similar to the CIV curve in this region of the leverage support. In contrast, there is little difference in  $\widehat{\text{CIV}}$  slope between medium and high leverage firms. The table suggests that firm heterogeneity primarily drives differences in CDS pricing among high leverage firms, but that the relative overpricing of deep OTM CDS, those for firms with below average leverage, is a robust feature of the data that does not appear explained by firm heterogeneity other than through leverage. It is likely that the remaining CIV smirk pattern is due to non-normalities, such as stochastic volatility or disaster risk, in the firm asset growth distribution. We investigate this possibility in the model of Section 5.

Note that correlation between leverage and the covariates in (2) can mean that the association between  $\widehat{\text{CIV}}$  and leverage is understated. We have also considered a version of  $\widehat{\text{CIV}}$  in which firm characteristics are first orthogonalized with respect to leverage before being included in (2). As one would expect, the resulting  $\widehat{\text{CIV}}$  smirk is quantitatively steeper than what we report here. Section 5 explores CIV heterogeneity in deeper detail and provides a more precise model-based account for how much of the smirk is driven by firm heterogeneity.

## 4.2 Dynamic Properties of CIV

Figure 4 plots snapshots of the CIV surface at specific points in time. The left panel shows the CIV surface as of December 29, 2006, which corresponds to the lowest month-end value of VIX during our sample. The right panel shows the surface on March 31, 2009, at the height of the financial crisis. During the crisis, asset implied volatility for ATM (high leverage) credits is higher and the

Figure 4: CIV SMIRK SNAPSHOTS



**Note.** Scatter plots of CIV (one-year and ten-year maturities) versus firm leverage at two points in time. The left panel corresponds to December 29, 2006 and the right panel to March 31, 2009. Lines are non-parametric curves fitted to CIV for each maturity.

moneyness slope is flatter than in the calm markets of 2006. While the curves are qualitatively similar in both figures, they suggest a role for time variation in the CIV surface. This subsection investigates dynamic properties of the surface in detail.

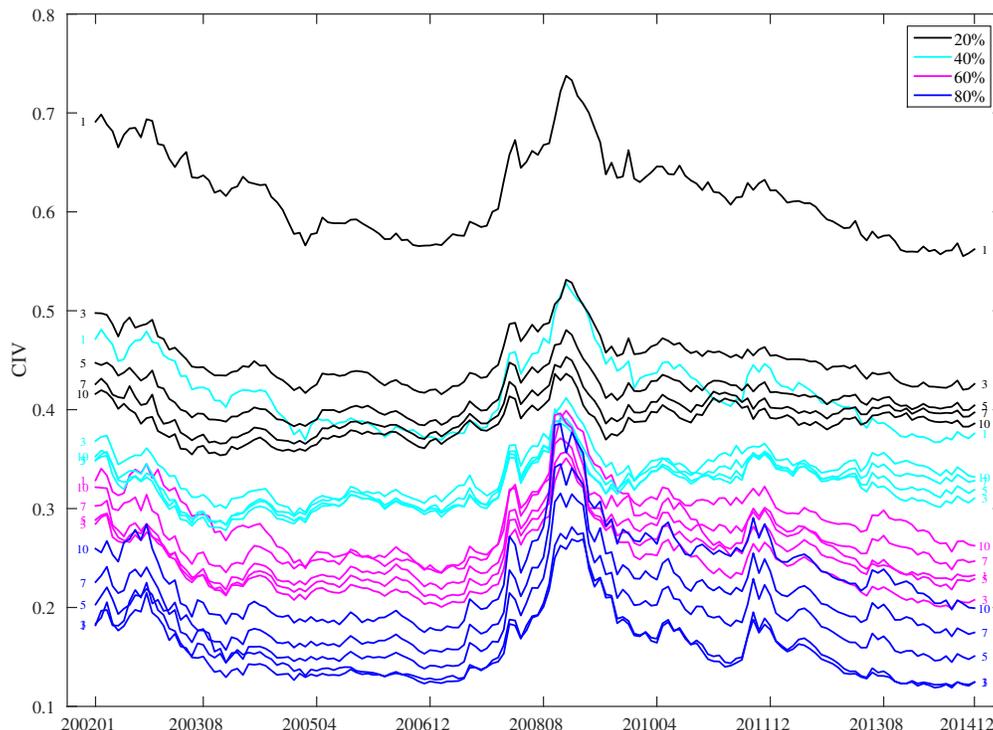
#### 4.2.1 Leverage-sorted Portfolios

Firm-level CDS comprise a rather severely unbalanced panel, so it is difficult to track the evolution of the CIV surface through time with a firm-level panel. Instead, we create and track portfolios of CDS that are designed to have fixed leverage, which proves especially convenient for visualizing fluctuations in the CIV surface.

We construct portfolio-level CIV each month by fitting a non-parametric curve through the scatter of CIV against leverage, just as in Figure 2, using only data for that month. CIV for constant-leverage portfolios is defined as the interpolated value of this curve at grid points of 20%, 40%, 60%, and 80% leverage. Portfolio-level CIV values from non-parametric curve fitting are local averages of CIV at each leverage grid point, where the weight on each firm’s CIV is decreasing in the distance between the grid point and the firm’s leverage.<sup>22</sup> We fit separate curves at each

<sup>22</sup>As in Figures 2, 3, and 4, the non-parametric curve is calculated with the matlab `smooth.m` function using the robust weighted least squares (“`rlowess`”) option and span parameter of 0.5, which defines the exact weighting scheme for local averages at each grid point/portfolio.

Figure 5: CREDIT IMPLIED VOLATILITY TIME SERIES (20 PORTFOLIOS)



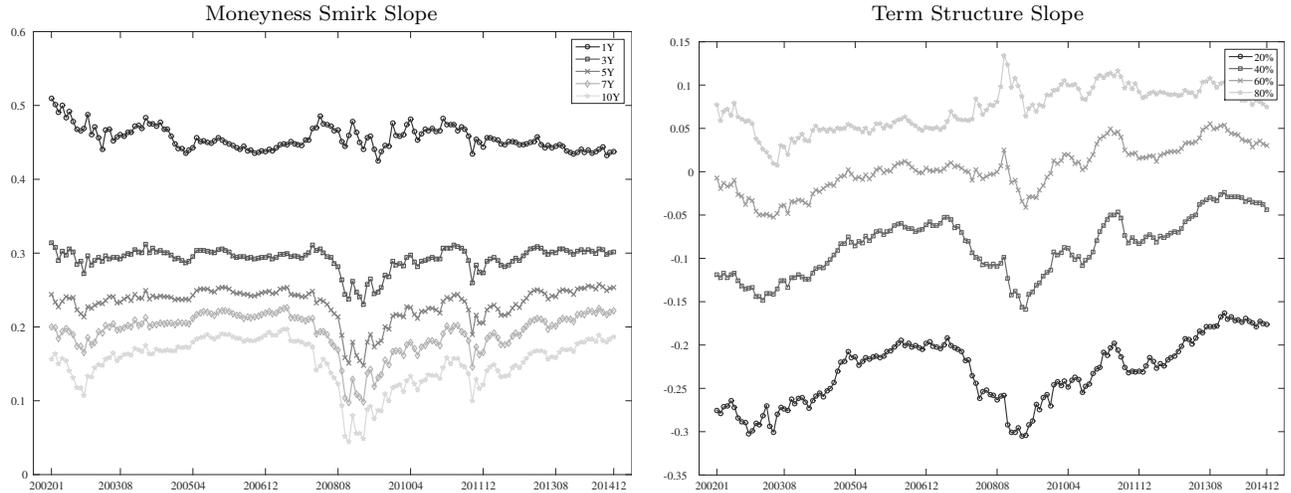
**Note.** CIV for 20 constant-leverage and constant-maturity portfolios. Colors distinguish the portfolio moneyness and portfolio maturity is printed beside each line.

maturity of 1, 3, 5, 7, and 10 years to produce a panel of monthly CIV for 20 constant-leverage and constant-maturity CDS portfolios.

The portfolio CIV series are plotted in Figure 5. All portfolios share substantial common fluctuations that manifest as shifts in the *level* of the surface. To summarize the evolution of the *shape* of the CIV surface, we plot the moneyness smirk and term structure slope over time. Steepness of the moneyness smirk each month is defined as the difference in CIV for 20% and 80% leverage portfolios. The left panel of Figure 6 plots time variation in smirk steepness for each maturity, and highlights the stability of the smirk pattern over time. In every month of our sample the moneyness slope is positive at every maturity.

Furthermore, the smirk is monotonically steeper for shorter maturity contracts, a relationship that again holds in each individual month. For one-year CDS, the smirk slope shows little time variation other than a slight downward trend. For other maturities, the smirk flattens during downturns, especially during the financial crisis. That is, while CIV rises for all portfolios during these

Figure 6: MONEYNES AND TERM STRUCTURE DYNAMICS



**Note.** Slope of the moneyness smirk (left panel) is defined as the difference in CIV for the 20% and 80% leverage portfolios and is plotted separately for each maturity. The term structure slope is defined as the difference in CIV for ten-year and one-year maturity portfolios and is plotted separately for each point on the leverage grid.

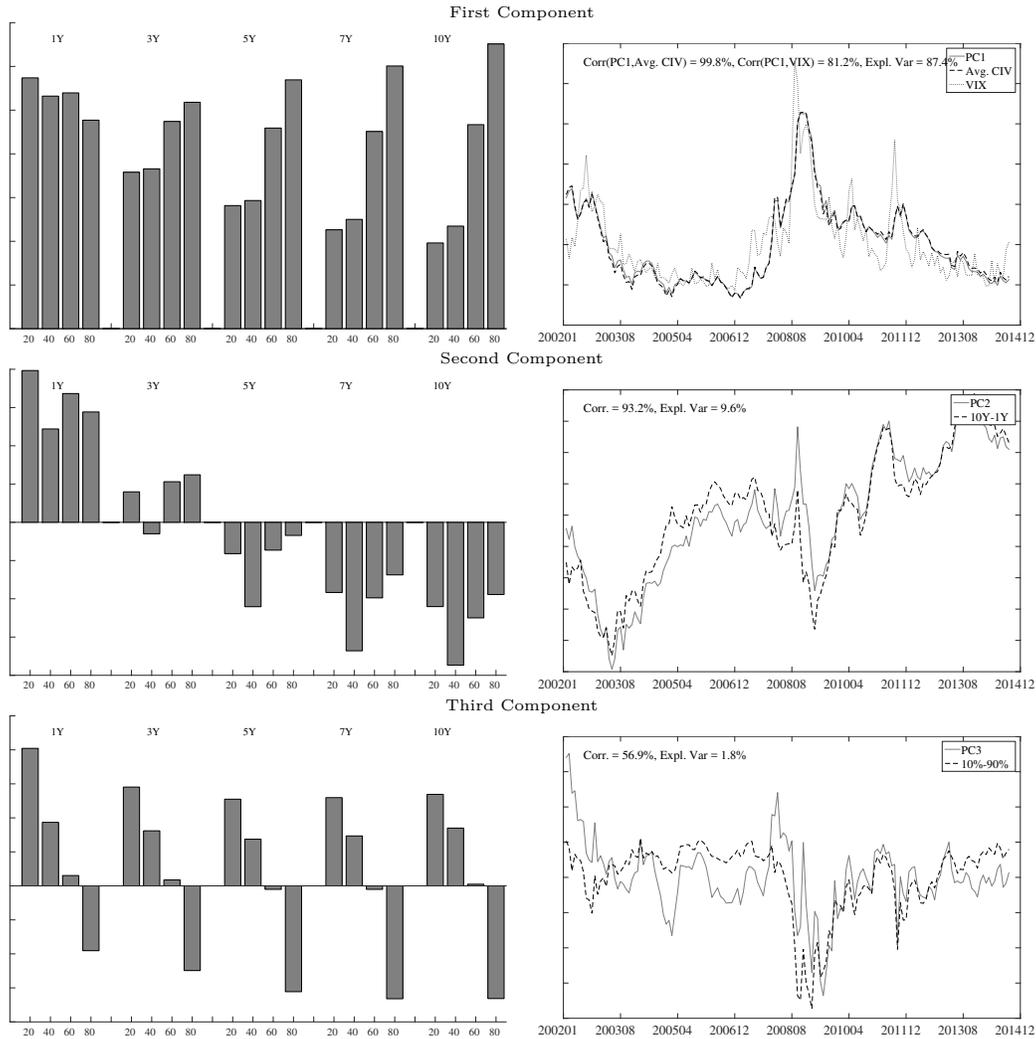
risky episodes (as seen from Figure 5), the relative price of CDS rises fastest for high leverage firms. Note, however, that time variation in the moneyness smirk is typically smaller than differences in the unconditional average smirk across maturities.

The right panel of Figure 6 shows dynamics in the CIV term structure slope, which is defined as the difference in the ten-year and one-year CIV, holding leverage fixed. In every month, the term structure of CIV is steeply downward sloping for deep OTM CDS, and is mildly upward sloping for ATM CDS, again showing that the basic shape of the unconditional surface holds at each point in time. And while there is clear cyclical variation in the term structure slope, this variation tends to be smaller than differences in the unconditional slope across leverage groups.

#### 4.2.2 CIV Factor Structure

The time series patterns in Figures 5 and 6 suggest a high degree of commonality in CIV fluctuations for portfolios sorted by leverage and maturity. We analyze the factor structure of the CIV surface using principal component (PC) analysis. We extract PCs from the panel of  $156 \times 20$  month-by-portfolio CIV observations. The five leading components explain 87.4%, 9.6%, 1.8%, 0.4%, and 0.3% of the panel variation in CIV, respectively, leading us to focus our analysis on the first

Figure 7: PRINCIPAL COMPONENTS OF CIV SURFACE



**Note.** Principal component analysis of CIV for 20 constant-leverage and constant-maturity portfolios. Loadings for the first three factors are shown on the left, and the first three factor time series are shown on the right.

three PCs. The left side of Figure 7 plots the loadings of the 20 leverage/maturity portfolios on each component. Loadings on the first PC are uniformly positive across portfolios, giving it the interpretation of a CIV level factor. Indeed, the top right panel shows that the first component shares a correlation of 99.8% with the average CIV across all portfolios. It also shows that the first component is closely related to the VIX index, with the two sharing a correlation of 81.2%.

Portfolio loadings on the second component align with portfolio maturity, indicating that this component is a term structure factor. It shares a correlation of 93.2% with the term structure slope (one-year CIV minus ten-year CIV, averaged across leverage groups). Loadings on the third

component line up with portfolio leverage, giving it the interpretation of a moneyness factor. When we compare this factor to the moneyness smirk slope (CIV at 20% leverage minus CIV at 80% leverage, averaged over maturities), we find they share a correlation of 56.9%.

Collectively, Figures 2 through 7 suggest that the vast majority of heterogeneity among CIV observations, both across individual CDS and over time, is associated with the leverage of the reference entity, the maturity of the CDS contract, and a small number of common time series factors (the most important of which behaves like aggregate market volatility).

Table 2 reports regressions that formalize this statement (using the firm panel as opposed to portfolios). In all columns, the dependent variable is  $CIV_{i,\tau,t}$ , where subscripts  $i$ ,  $\tau$ , and  $t$  again denote reference firm, contract maturity, and month, respectively. The specification reported in the first column is

$$CIV_{i,\tau,t} = \delta_0 + \delta_1 \text{Leverage}_{i,t} + \epsilon_{i,\tau,t}, \quad (3)$$

and summarizes the extent to which variation in CIV is explained by leverage alone. Leverage is highly significant and explains 48.6% of the panel variation in CIV. The coefficient estimate of  $-0.397$  is interpretable as the average slope of the CIV smirk—a change in leverage from zero to 100% is associated with a decrease in CIV of 40% per year. This is of course pooled over all observations and ignores non-linearities in the empirical smirk.

The second column modifies the previous regression to include an additional term equal to  $\delta_{3,i} \text{VIX}_t$ . This controls for CIV fluctuations that arise from a common aggregate factor in CDS prices, in this case proxied by the VIX index. Rather than using month fixed effects, we choose this specification because it accommodates heterogeneity in firm exposure to the common factor via the  $\delta_{3,i}$  coefficient and is motivated by the structural model we propose in Section 5. Accounting for a single common factor in CIV raises the regression  $R^2$  to 67.5%.

We next modify the regression to include the effects of contract maturity,

$$CIV_{i,\tau,t} = \delta_0 + \delta_1 \text{Leverage}_{i,t} + \text{Maturity FE} + \delta'_2 \text{Maturity FE} \times \text{Leverage}_{i,t} + \epsilon_{i,\tau,t}.$$

Maturity fixed effects capture differences in the overall level of CIV across the term structure, and interacting leverage with maturity dummies captures differences in the moneyness slope at different

Table 2: DETERMINANTS OF CIV

	(1)	(2)	(3)	(4)	(5)	(6)
Lev.	-0.397*	-0.402*	-0.201*	-0.206*	-0.130*	-0.191*
1Y			0.311*	0.311*	0.290*	0.311*
3Y			0.102*	0.101*	0.092*	0.101*
5Y			0.050*	0.049*	0.043*	0.049*
7Y			0.022*	0.022*	0.020*	0.022*
1Y×Lev			-0.529*	-0.530*	-0.497*	-0.530*
3Y×Lev			-0.246*	-0.245*	-0.231*	-0.246*
5Y×Lev			-0.138*	-0.137*	-0.129*	-0.137*
7Y×Lev			-0.070*	-0.070*	-0.067*	-0.070*
Size					-0.659*	-0.729*
Vol.					0.262*	0.128*
Skew.					-0.187*	-0.141*
Kurt.					-0.028*	-0.023*
Beta					-0.025	0.870*
AA					-0.003	-0.004
BBB					-0.001	0.008
BB					0.017*	0.026*
Cons. Prod.					0.005	0.002
Cons. Svc.					0.003	0.001
Energy					-0.002	-0.017
Financials					-0.027	-0.016
Hlth.					0.014	0.021
Indust.					-0.011	-0.010
Tech.					0.023	0.071*
Telcom. Svc.					-0.001	0.001
Util.					-0.026	-0.025*
VIX	No	Yes	No	Yes	Yes	Yes
$N$	253,410	253,410	253,410	253,410	151,259	253,188
$R^2$	0.486	0.675	0.650	0.840	0.878	0.869

**Note.** Dependent variable is firm level CIV. VIX row indicates whether the VIX index enters as a common explanatory variable (with firm-specific loading). In Column 5, realized risk measures are computed from total asset returns, while in Column 6 they are estimated from equity returns. Omitted dummies are 10-year maturity, Materials sector, and credit rating A. Coefficients on Size, Skew., Kurt., and Beta are multiplied by 100. Standard errors are clustered by month and sector. Asterisk indicates significant at the 1% level or better.

points in the term structure. This specification is equivalent to running regression (3) separately for each maturity. We consider a version of this regression excluding the  $\delta_{3,i}VIX_t$  term (Column 3) and including it (Column 4). Accounting for the effect of maturity on CIV increases the regression  $R^2$  to 65.0% and 84.0% in Columns 3 and 4, respectively.

The final two columns modify the previous specifications to allow for additional firm-level characteristics including size, riskiness, industry sector, and credit rating:

$$\begin{aligned}
CIV_{i,\tau,t} = & \delta_0 + \delta_1 \text{Leverage}_{i,t} + \text{Maturity FE} + \delta_2' \text{Maturity FE} \times \text{Leverage}_{i,t} + \delta_{3,i} VIX_t \\
& + \delta_4' [\text{Size}_{i,t}, \text{Vol}_{i,t}, \text{Skew}_{i,t}, \text{Kurt}_{i,t}, \text{Beta}_{i,t}]' + \text{Sector FE} + \text{Rating FE} + \epsilon_{i,\tau,t}.
\end{aligned}$$

Volatility, skewness, kurtosis, and market beta in Column 5 are measured from asset returns (described in Appendix A), and are measured from equity returns in Column 6. This regression is designed to quantify the fraction of CIV variation that is driven by firm characteristics but that cannot be accounted for by leverage, maturity, and exposure to aggregate risk. The regression  $R^2$  in these specifications rises from 84.0% in Column 4 to 87.8% and 86.9% in Columns 5 and 6, indicating little marginal explanatory power from firm characteristics beyond leverage. The regression coefficient on leverage is lower in Column 5 due to positive correlation between leverage and asset volatility.

### 4.3 CIV and Risk Premia

Thus far our CIV analysis has focused on CIV levels. The analysis in Figure 7 shows that CIV of leverage and maturity sorted portfolios are all but perfectly explained with two or three common factors. Perhaps more interestingly, the regressions in Table 2 indicate that leverage and maturity are among the most important dimensions of heterogeneity for understanding relative credit price levels. This naturally invites the question: Do average returns of a CDS contract also align with its leverage and maturity? This is a sensible hypothesis if CDS risk exposures also align with contract moneyness and maturity.

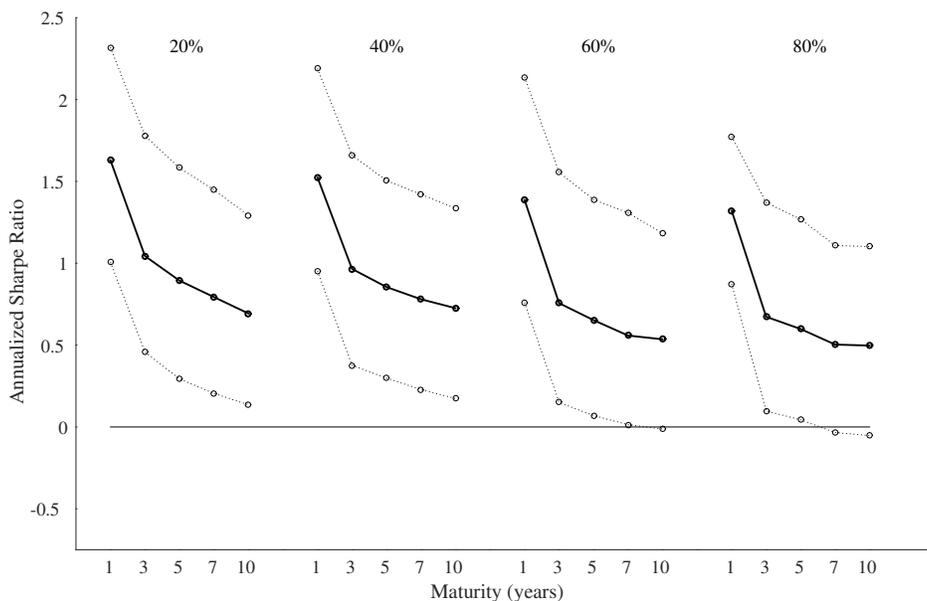
We investigate this question by constructing realized returns on the same constant-leverage and constant-maturity portfolios studied above. As with portfolio CIV, portfolio returns are calculated as local averages of individual CDS returns at each maturity, where the portfolio weights each month are inversely related to the distance between a contract’s leverage and a given constant-leverage grid point.<sup>23</sup>

Figure 8 plots annualized Sharpe ratios of monthly returns on the 20 CDS portfolios (along with 95% bootstrap confidence intervals). These are returns to a CDS seller and so represent risk premia that accrue to an insurance provider. The answer is yes, differences in compensation for selling CDS closely align with CDS leverage and maturity. The term structure of CDS Sharpe ratios is steeply downward sloping, reaching as high as 1.6 for one-year CDS and falling to as low as 0.5 at ten years. In the moneyness dimension, Sharpe ratios are highest for deep OTM CDS and

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<sup>23</sup>The CDS return definition is given in Appendix A. Portfolio average returns are again calculated with the matlab `smooth.m` function with the “`rlowess`” option and span parameter of 0.5. This defines the exact weighting scheme for average returns in each leverage portfolio.

Figure 8: SHARPE RATIOS OF LEVERAGE AND MATURITY PORTFOLIOS



**Note.** The solid line shows annualized Sharpe ratios from monthly CDS returns of constant-leverage and constant-maturity portfolios. Dotted lines are the associated 95% bootstrap confidence intervals.

gradually decline with leverage.<sup>24</sup>

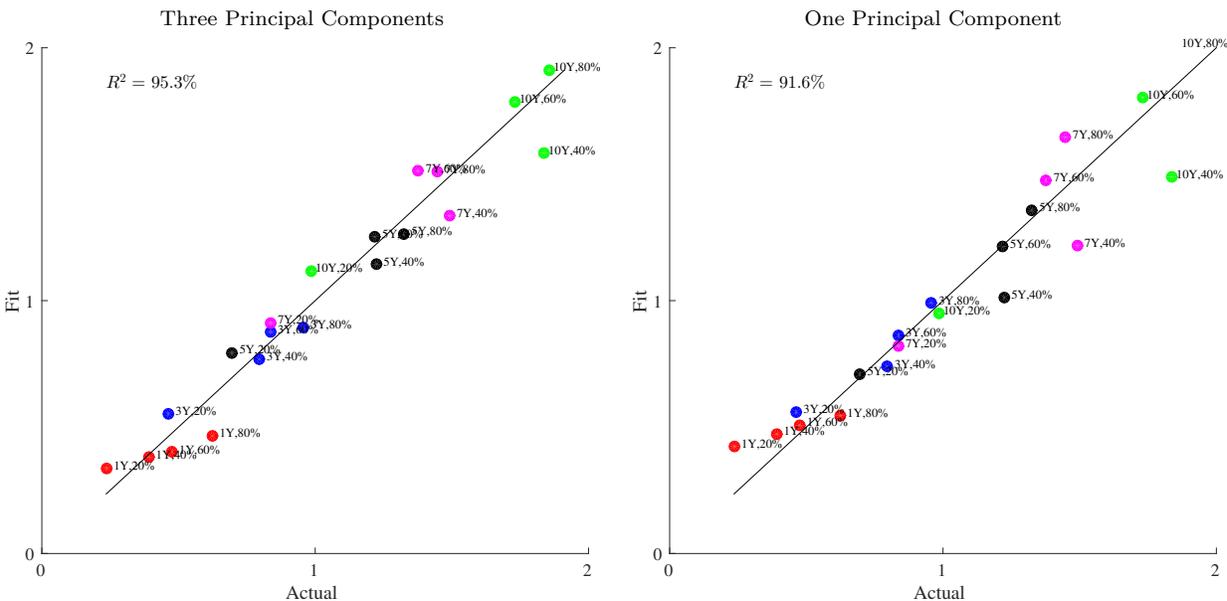
Furthermore, differences in risk compensation across CDS are explained by their differences in exposure to aggregate CIV shocks. We show this by constructing pricing factors that are shocks to the first three principal components of the CIV surface.<sup>25</sup> We evaluate the ability of factors to price mean CDS returns using two-pass regression; first estimating betas from full sample time series regressions of each portfolio's return onto the shocks, then running a single second-pass cross section regression of average portfolio returns on first-pass betas.

Figure 9 plots the predicted average portfolio returns from the second stage regression against actual values. The left panel is based on a three-factor model (from the first three components), and the right panel uses only the first principal component. The fits show that differences in risk premia across portfolios are essentially fully explainable with differences in their exposure to changes in the surface. Using the level, moneyness, and term structure factors jointly in a three-factor model

<sup>24</sup>The finding that short-maturity CDSs have higher Sharpe ratios is consistent with [Palhares \(2013\)](#), who shows that short positions in one-year CDS suffered worse losses than five and ten-year contracts during crisis, consistent with the higher compensation we find for selling short-dated contracts.

<sup>25</sup>We define shocks as first differences of the PCs, but results are nearly identical for other shock definitions such as residuals from a vector autoregression using the first three PCs.

Figure 9: AVERAGE RETURN FITS



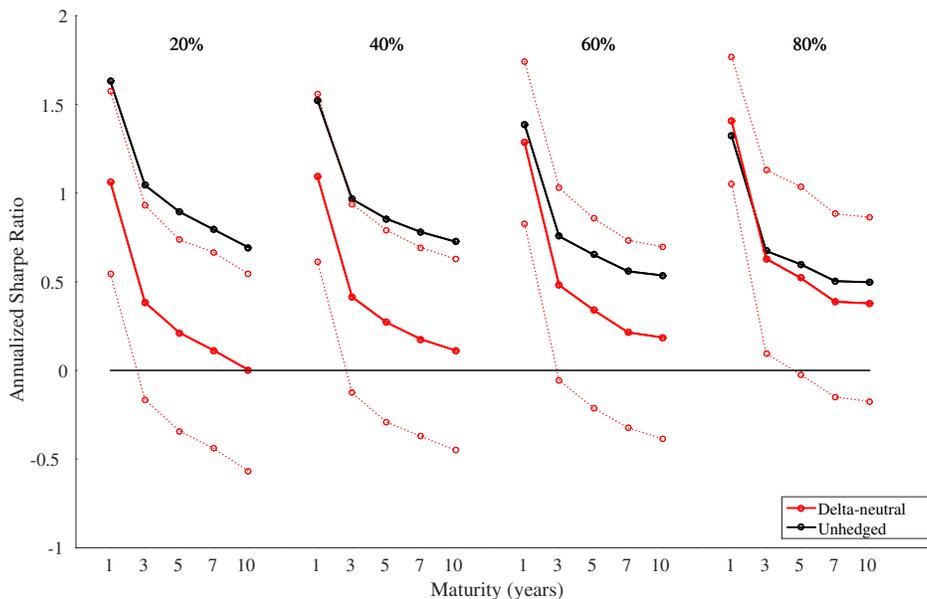
**Note.** Second-pass fitted versus actual average returns for the 20 constant-leverage and constant-maturity portfolios from three-factor and one-factor models in which factors are defined as changes in the leading principal components of the CIV surface. Axes are annualized percentage returns, and the second-pass regression  $R^2$  is reported in the upper left.

delivers pricing errors that are uniformly close to zero with a cross section  $R^2$  of 95.3%. Nearly all of this explanatory power comes from exposure to the level shock, as shown in the right panel of the figure, where the first factor alone produces almost identical fits to the three-factor model and results in a cross section  $R^2$  of 91.6%.

These facts are related to [Palhares \(2013\)](#), who also documents that CDS Sharpe ratios are decreasing in maturity. We expand on that analysis by providing a more complete view of CDS prices and risk premia in the dimensions that appear most empirically relevant: leverage and maturity. [Palhares \(2013\)](#) also proposed an empirical pricing model to explain the term structure of expected returns using two factors, the average CDS return and the CDS return on a short-maturity/long-maturity spread portfolio. We tie the behavior of credit risk premia directly to the CIV surface and find that a single CIV factor successfully explains the cross section differences in average CDS returns.<sup>26</sup>

<sup>26</sup>This result is also related to [Fama and French \(1993\)](#) who find that average returns on maturity-sorted Treasury bond portfolios and ratings-sorted corporate bond portfolios can be explained with a two-factor model (the factors being the term spread and the default spread), as well as to [Frazzini and Pedersen \(2012\)](#) who find option return premia with similar moneyness and maturity patterns.

Figure 10: SHARPE RATIOS OF DELTA-HEDGED CDS PORTFOLIOS



**Note.** The solid red line shows annualized Sharpe ratios from monthly delta-hedged CDS returns of constant-leverage and constant-maturity portfolios. Dotted red lines are the associated 95% bootstrap confidence intervals. For comparison, the black line overlays the Sharpe ratio of unhedged CDS returns from Figure 8.

Lastly, we decompose the credit risk premium measured from CDS portfolios into two components, each understandable from the interpretation of a CDS as an option contract. One is an asset risk premium component, closely related to the standard equity risk premium, arising from the fact that a CDS (or “put on assets”) has a mechanical loading (or “delta”) on overall asset growth in the economy. Second, the embedded put option gives CDS returns a mechanical loading (or “vega”) on shocks to asset volatility, which earns it a variance risk premium.

To analyze the portion of total credit risk compensation that comes from bearing variance risk, we construct delta-hedged CDS portfolios. These begin with a short position in CDS and take an offsetting position in the firm’s equity:

$$R_{\text{hedge},i,t} = R_{\text{cds},i,t} - \Delta_i \times R_{\text{equity},i,t}.$$

We construct the hedge ratio,  $\Delta_i$ , using a full sample regression of CDS returns for portfolio  $i$  onto the underlying equity return (using the same weighting scheme to construct portfolio equity returns as for CDS returns). The standard way of constructing a delta-hedged portfolio in the

options literature is to calculate  $\Delta_i$  using the Black-Scholes model and use this to hedge out the underlying equity return. The analogue in our setting is to calculate  $\Delta_i$  from the Merton model and hedge out the underlying asset return. We have thus taken a more statistically oriented approach of hedging with the underlying equity and use the empirical hedge ratio, though our results are qualitatively the same if we construct (approximate) asset returns as in Appendix A and hedge using the Merton-implied  $\Delta_i$ .<sup>27</sup> Properly speaking, a delta-hedged option position earns compensation for exposure to all higher moment risk, not just variance. But this is true in most variance risk premium calculations and we continue to use variance terminology in accordance with previous literature.

Sharpe ratios of delta-hedged CDS portfolios are plotted in Figure 10. We also plot the 95% bootstrap confidence interval for each portfolio, and overlay on this the raw CDS Sharpe ratio from Figure 8 for comparison. The basic pattern of Sharpe ratios with respect to moneyness reverses, and is now increasing in moneyness, compared to a decreasing pattern for the unhedged CDS position. Other than this difference, delta-hedged Sharpe ratios are qualitatively and quantitatively similar to unhedged CDS. In particular, the steep negative term structure pattern is entirely preserved.<sup>28</sup> This suggests that the component of the Sharpe ratio associated with the standard equity premium only varies in the moneyness dimension, and that the term structure of credit risk premia is dominated by the variance risk premium.

## 5 A Structural Model of Credit Risk

The options literature has sought, and has been successful in isolating, models to accurately describe option pricing phenomena. This research has targeted the implied volatility surface as the object of interest based on its convenience for summarizing the joint behavior of many option prices on a common scale. The patterns in credit-implied volatility closely resemble those of option-implied volatility.<sup>29</sup> CIV is higher for OTM CDS, just as OIV is higher for OTM puts, holding other

<sup>27</sup>Our delta-hedged CDS strategy is similar in spirit to the capital structure arbitrage strategy studied in Duarte, Longstaff and Yu (2007).

<sup>28</sup>This fact bears a close resemblance to the term structure of equity variance risk premia studied by Dew-Becker et al. (2015), and is consistent with a more general pattern in the term structure of Sharpe ratios for many asset classes pointed out by Van Binsbergen and Koijen (2015).

<sup>29</sup>See Andersen, Fusari and Todorov (2015) for a detailed description of the implied volatility surface of S&P 500 index options.

features of the underlying and the contract fixed. In both credit and option markets, the implied volatility surface is driven by a small number of common factors. The most important factor corresponds to persistent shifts in the overall level of the CIV surface, and this factor closely tracks aggregate market volatility. And in both cases, other important factors capture time variation in the steepness of the moneyness smirk and shape of the term structure. Recognizing the similarities between CIV and OIV surfaces enables us to improve our understanding of CDS pricing by drawing on prior work modeling equity options.

In this section we propose a model to describe the joint behavior of CDS for all firms and all maturities. Three ingredients are necessary to match the CIV patterns described in the preceding sections. First, we pursue a specification in which asset growth of individual firms is driven by exposure to aggregate asset growth and an idiosyncratic growth shock. This assumption of a factor structure in growth rates has strong empirical support and is one means of tying together behavior of CDS spreads across firms. Second, motivated by the options literature, we embed stochastic volatility and jumps in the aggregate asset growth process. This produces the non-normalities in asset growth required to generate a moneyness smirk. Dynamics in aggregate volatility and jump risk produce common fluctuations in firms' CDS prices. Third, idiosyncratic variance and jump risk dynamics are closely aligned with the same state variables that drive aggregate asset growth, a pattern well documented in prior literature (see [Herskovic et al. \(2015\)](#) and [Kelly and Jiang \(2014\)](#)). By virtue of firms' common exposure to aggregate risks, CDS dynamics of all firms and at all maturities inherit the dynamics of a few common state variables, capturing the persistent level shifts and twisting motion of the CIV surface. We find that at least two factors are necessary to generate the distinct time-series patterns we see for the level of the surface, the moneyness slope, and the term structure slope.

In the language of the credit risk literature, our model is structural, in the sense that it specifies a process for aggregate asset growth and derives cross-equation restrictions for pricing CDS across individual firms and across maturities based on firms' sensitivities to aggregate growth. However, we abstract from specifying deep technologies and preferences and do not model the optimizing behavior of firms or consumers. That is, we model exogenous asset growth processes under the risk-neutral, or  $\mathbb{Q}$ , measure, and in this sense our model is partly reduced form. These abstractions are necessary for keeping the estimation problem tractable while emphasizing the role of firm

heterogeneity and state dependence in credit price behavior. The goal of this section is to demonstrate that a parsimonious no-arbitrage model with a unified description of asset growth for all firms delivers highly accurate fits of CDS prices across a wide range of credit risk and at all points throughout the credit cycle.

## 5.1 Model

We analyze four models of the joint asset value dynamics of all firms in the economy. The models are of gradually increasing complexity—here we present the most general version that nests the other three specifications.

In the most general model we consider, aggregate asset growth follows a three state variable process. The choice of at most three states is motivated by the PCA results in Section 4. The first two states govern stochastic growth rate volatility. One process,  $v_{1,t}$ , captures patterns in volatility that persist over the business cycle and other low frequencies. The second state,  $v_{2,t}$ , captures higher frequency movements in volatility that mean revert within months. We allow for a two volatility specification based on a large literature documenting fast-moving and slow-moving components of volatility.<sup>30</sup> We allow for a third state variable that governs jumps in aggregate asset value and allows for stochastic arrival intensity. This injects more excess kurtosis in the asset growth distribution and is motivated by the steep CIV moneyness smirk.<sup>31</sup> The three state variables govern the aggregate asset growth process according to

$$\frac{dA_{m,t}}{A_{m,t}} = rdt + \sqrt{v_{1,t}}dW_t^{m,1} + \sqrt{v_{2,t}}dW_t^{m,2} + ((e^{-q_m} - 1) dJ(\lambda_t) - \lambda_t \xi_m), \quad (4)$$

where  $(W_t^{m,1}, W_t^{m,2})$  are independent Brownian motions and the third term is a standard Poisson mixture of normals jump specification. Variances are CIR processes (Cox, Ingersoll and Ross,

<sup>30</sup>Bates (2000); Andersen, Fusari and Todorov (2015); Johnson (2012) document the need for two volatility factors to match the diffusive component of aggregate equity market returns. Furthermore, because CDS are derivatives on underlying asset values, they are also closely connected to macroeconomic volatility. Jurado, Ludvigson and Ng (2015) also document a low frequency component in macroeconomic volatility with a half-life of over four years.

<sup>31</sup>A large literature documents that jumps are crucial for producing sufficient leptokurtosis to match the OIV moneyness smirk. See, for example, Bakshi, Cao and Chen (1997); Broadie, Chernov and Johannes (2007).

1985),

$$\begin{aligned} dv_{1,t} &= \kappa_{v_1} (\theta_{v_1} - v_{1,t}) dt + \sigma_{v_1} \sqrt{v_{1,t}} dW_t^{v_1} \\ dv_{2,t} &= \kappa_{v_2} (\theta_{v_2} - v_{2,t}) dt + \sigma_{v_2} \sqrt{v_{2,t}} dW_t^{v_2}, \end{aligned}$$

and are allowed to mean revert at different rates ( $\kappa$ ), to different unconditional levels ( $\theta$ ), with different volatility of volatility ( $\sigma$ ).<sup>32</sup> The arrival intensity of jumps is allowed to depend on both variance processes as well as on third state variable  $z_t$  that independently drives jump risk:

$$\lambda_t = a(v_{1,t} + v_{2,t}) + z_t.$$

The independent component of the jump intensity also follows a CIR process,

$$dz_t = \kappa_z (\theta_z - z_t) dt + \sigma_z \sqrt{z_t} dW_t^z.$$

Finally the distribution of jump sizes is  $q_m \sim N(\mu_{q_m}, \sigma_{q_m}^2)$ , and the jump compensator is  $\xi_m = e^{-\mu_{q_m} + .5\sigma_{q_m}^2} - 1$ .

Next, asset growth rates for individual firms are tied together by exposure to the aggregate growth rate distribution. In particular, asset growth of firm  $i$  has a systematic component tied to aggregate growth as well as an idiosyncratic growth rate:

$$\frac{dA_{i,t}}{A_{i,t}} = rdt + \beta_i \left( \frac{dA_{m,t}}{A_{m,t}} - rdt \right) + \sqrt{v_{i,t}} dW_t^i + ((e^{-q_i} - 1) dJ(\lambda_t) - \lambda_t \xi_i). \quad (5)$$

The idiosyncrasy possesses diffusive and jump components. We tightly link the conditional distribution of idiosyncratic growth to that of aggregate growth.<sup>33</sup> In particular, we assume that idiosyncratic stochastic volatility is perfectly correlated with aggregate stochastic volatility,

$$v_{i,t} = v_i + \gamma_i(v_{1,t} + v_{2,t}),$$

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<sup>32</sup>We also allow for a correlation between variance shocks and asset growth. This enters through a correlation of  $\rho_{v_j}$  between  $dW_t^{v,j}$  and  $dW_t^{m,j}$ ,  $j = 1, 2$ .

<sup>33</sup>This assumption is in part motivated by [Campbell and Taksler \(2003\)](#) who emphasize the role of idiosyncratic risk in explaining credit spreads.

Table 3: PORTFOLIO PARAMETER ESTIMATES

	20%	40%	60%	80%
$\beta_i$	1.490	1.288	1.033	0.798
Asset gr. $R^2$ (%)	24.8	28.6	34.0	43.2
$\gamma_i$	1.895	1.300	0.866	0.453
$v_i$	0.019	0.010	0.004	0.002
Asset var. $R^2$ (%)	77.2	84.7	88.9	88.8

**Note.** Parameter estimates for constant-leverage portfolios.

and we assume that idiosyncratic jumps share the same distribution as aggregate jumps,  $q_i \sim N(\mu_{q_m}, \sigma_{q_m}^2)$ , and arrive with the same intensity  $\lambda_t$ .

This specification for firm-level asset growth is empirically motivated. First, asset growth of individual firms is highly correlated with aggregate growth. To demonstrate this, we run separate time series regressions of monthly asset returns (described in Appendix A) for each firm in our sample on aggregate asset growth (asset-weighted average returns across all firms). Table 3 reports the average beta and regression  $R^2$  within each constant-leverage portfolio. Portfolio betas range from 0.79 to 1.49 and the  $R^2$  is between 24.8% and 43.2%. Second, idiosyncratic firm volatility is highly correlated with aggregate volatility. For each leverage portfolio, we regress its realized monthly asset variance on aggregate asset variance. The intercept (denoted  $v_i$ ) and slope coefficient (denoted  $\gamma_i$ ) are also reported in Table 3, as well as the regression  $R^2$ . The aggregate asset variance process describes between 77.2% and 88.9% of the variation in idiosyncratic variance. Both sets of regressions support the firm-level asset specification in (5). Third, our choice to set idiosyncratic jump risk equal to aggregate jump risk is motivated by the fact that fluctuations in firm-level tail risks are strongly correlated in the cross section (Kelly and Jiang, 2014). But, in contrast to idiosyncratic volatility comovement, tail risk comovement is difficult to precisely quantify. Thus, our assumption captures the basic empirical fact without additional (and hard to estimate) parameterization of the model.

In our model analysis we also consider three smaller models nested in this main “2 Vol, 1 Jump” specification. The simplest, which we refer to as “1 Vol, 0 Jump,” includes a single stochastic volatility process and shuts off the second volatility and the jump. We also consider a jump-only specification, “0 Vol, 1 Jump,” that shuts down both volatilities. Lastly, we study the “1 Vol, 1 Jump” specification that shuts off only the second volatility.

## 5.2 Estimation

We derive the CDS pricing formula for this model in Appendix B using the framework of [Duffie, Pan and Singleton \(2000\)](#). The formula evaluates spreads predicted by our model for any CDS given the leverage of the reference entity, the maturity of the CDS contract, and given model parameters. In the most general version of the model there are 14 parameters for the aggregate asset growth process,

$$\Theta_m = (\kappa_{v_1}, \theta_{v_1}, \sigma_{v_1}, \rho_{v_1}, \kappa_{v_2}, \theta_{v_2}, \sigma_{v_2}, \rho_{v_2}, \kappa_z, \theta_z, \sigma_z, \mu_{q_m}, \sigma_{q_m}, a)$$

and three parameters for each reference entity,  $\Theta_i = (\beta_i, v_i, \gamma_i)$ .

We estimate the model using data for the 20 constant-leverage and constant-maturity CDS portfolios. These 20 portfolios only require modeling four distinct “reference entities,” one for each leverage group (the model must match behaviors of all five maturity bins while respecting a single asset process for each reference entity). Thus our general specification includes a total of 26 static parameters.

We split the estimation problem into two steps. In the first step, we set the betas and idiosyncratic volatility parameters ( $\Theta_i$ ) to the regression estimates in Table 3. Because these are estimated from realized asset growth data, they represent parameter values under the physical, or  $\mathbb{P}$ , measure. We follow the standard consistency conditions imposed in the literature and equate betas and idiosyncratic risks under  $\mathbb{P}$  and  $\mathbb{Q}$ .<sup>34</sup> There are two caveats to this approach. First, we only use the variance regressions to pin down the diffusive idiosyncratic variance parameters while, properly speaking, asset growth variance depends on both diffusive and jump risks. We make this approximation for tractability. Second, as noted above, we assume that the idiosyncratic jump distribution is identical with (but independent of) the aggregate jump distribution. This is, again, an assumption for tractability.

We estimate the remaining parameters,  $\Theta_m$ , directly from portfolio CDS spreads. The estimation objective is to minimize the squared distance between actual spreads,  $S_{i,\tau,t}$ , and model-predicted spreads,  $\hat{S}(\Theta_m, \hat{\Theta}_i, X_t, \tau)$ , where  $i$  indexes the underlying entity’s leverage,  $\tau$  the contract maturity,  $t$  the month, and  $X_t$  the vector of latent states. The scale of spreads varies substantially across leverage and maturity portfolios. We normalize pricing errors for each portfolio by the time

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<sup>34</sup>This is similar to the approach by [Ang and Longstaff \(2013\)](#).

Table 4: MODEL COMPARISON

	No-arbitrage				Unrestricted		
	1 Vol, 0 Jump	0 Vol, 1 Jump	1 Vol, 1 Jump	2 Vol, 1 Jump	PCA1	PCA2	PCA3
$R^2$ (%)	10.6	70.0	77.9	83.5	88.6	98.6	99.4
Parameters	4 + 12	5 + 12	10 + 12	14 + 12	40	60	80
States/factors	1	1	2	3	1	2	3

**Note.** Comparison of CDS spread fits from structural no-arbitrage models and PCA with one to three factors. We report the  $R^2$  with respect to spreads and the parameterization of each model. For parameter counts in the no-arbitrage models, “+12” highlights that 12 parameters are estimated in a preliminary step from physical CDS returns. Those 12 estimates are not, however, free parameters in the structural model estimation. It is therefore most natural to compare, for example, 10 parameters in “1 Vol, 1 Jump” with 60 parameters in “PCA2.”

series standard deviation of the portfolio’s observed spreads. This places all portfolios on a level playing field in terms of their contribution to the objective function.<sup>35</sup> The optimization problem is therefore

$$\min_{\Theta_m, \{X_t\}} \sum_{i, \tau, t} \left( \frac{S_{i, \tau, t} - \hat{S}(\Theta_m, \hat{\Theta}_i, X_t, \tau)}{\text{Std}(S_{i, \tau, t})} \right)^2. \quad (6)$$

We take an iterative implied state estimation approach along the lines of Pan (2002). First, we guess a value for the static model parameters. Then, holding these parameters fixed, we estimate the latent states ( $X_t$ ) by minimizing pricing errors for all portfolios in month  $t$ . Given the estimated values of  $X_t$ , we search for a new set of static parameters that lowers the objective function, and iterate this procedure to convergence.<sup>36</sup>

### 5.3 Results

Table 4 summarizes model fits in terms of total  $R^2$  for spreads on all 20 portfolios. The structural models are organized under the “No-arbitrage” heading, referring to their enforcement of cross-equation pricing restrictions across leverage and maturity, and are ordered from least parameterized to most parameterized. The immediate take-away from this analysis is that jumps are the crucial

<sup>35</sup>Another natural normalization is to minimize errors in CIV, as asset implied volatilities are more directly comparable across portfolios than are spreads. We check that this produces nearly identical results by using optimized parameters from the spread-based objective as starting values in the CIV-based optimization. We choose to focus our analysis on spreads as this objective requires less computation time by avoiding the CIV inversion for each data point in each numerical iteration.

<sup>36</sup>Our estimation uses a Matlab optimization routine that alternates between simulated annealing, direct search, and gradient descent to identify globally optimal parameter estimates.

modeling element for matching spreads on leverage and maturity portfolios. Stochastic volatility alone (“1 Vol, 0 Jump”) cannot explain spreads, delivering an  $R^2$  of only 11%. But a single jump process (“0 Vol, 1 Jump”), which costs only one more parameter than the volatility specification, delivers an  $R^2$  of 70.0%. Adding a second or third state variable produces further fit gains but requires richer parameterization (an entire time series of state realizations must be estimated for each additional state variable). “1 Vol, 1 Jump” increases the  $R^2$  to 77.9% but requires 5 additional static parameters and one additional state series, and “2 Vol, 1 Jump” increases the  $R^2$  again to 83.5% but requires another four static parameters and a third state series. In terms of model comparison, the “1 Vol, 0 Jump” is easily rejected in favor of the alternatives. An  $F$ -test rejects “0 Vol, 1 Jump” in favor of “1 Vol, 1 Jump” with a  $p$ -value of essentially zero. The “1 Vol, 1 Jump” specification is only marginally rejected in favor of “2 Vol, 1 Jump” with a  $p$ -value of 0.04.

We also compare the structural model to the fits from PCA. The PCA model takes the form

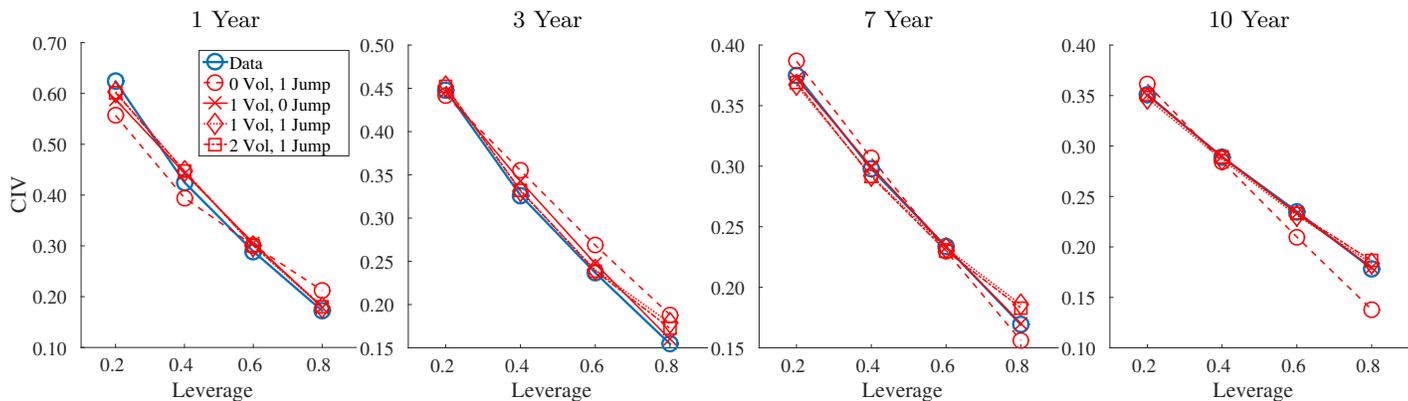
$$S_{i,m,t} = \alpha_{0,i,m} + \alpha'_{1,i,m}F_t + \omega_{i,m,t}.$$

The dependent variable is the panel of spreads for the 20 portfolios. The subscript  $i$  describes the portfolio leverage,  $m$  describes its maturity, and  $t$  the month of the observation. There are two key differences between the structural and PCA specifications. The first difference is that PCA is a linear specification while the structural framework is non-linear. The second difference is that PCA is an unrestricted model—it does not impose cross-equation pricing restrictions implied by no-arbitrage. In particular, for any given underlying  $i$ , the factor loadings of  $i$ ’s CDS spreads at different maturities ( $\gamma_{1,i,m}$ ) are in no way linked together; however, in a no-arbitrage setting, the state sensitivity of spreads across maturity must all be determined by the same deep model parameters. The unrestricted nature of PCA allows for substantially more parameters than a no-arbitrage specification with the same number of factors.

Despite the much richer parameterization of PCA, the structural specifications with jumps are competitive in terms of  $R^2$ . Because our model uses many fewer parameters, its comparative fit success must be driven by cross-equation restrictions and non-linearities embedded in the no-arbitrage accurately capturing the joint behavior of CDS with different moneyness and maturity.

Next, we evaluate the fits of the model by studying its ability to match the unconditional

Figure 11: STRUCTURAL MODEL MONEYNESS SLOPE



**Note.** Actual and model-predicted moneyness smirk for 20 constant-leverage and constant-maturity portfolios.

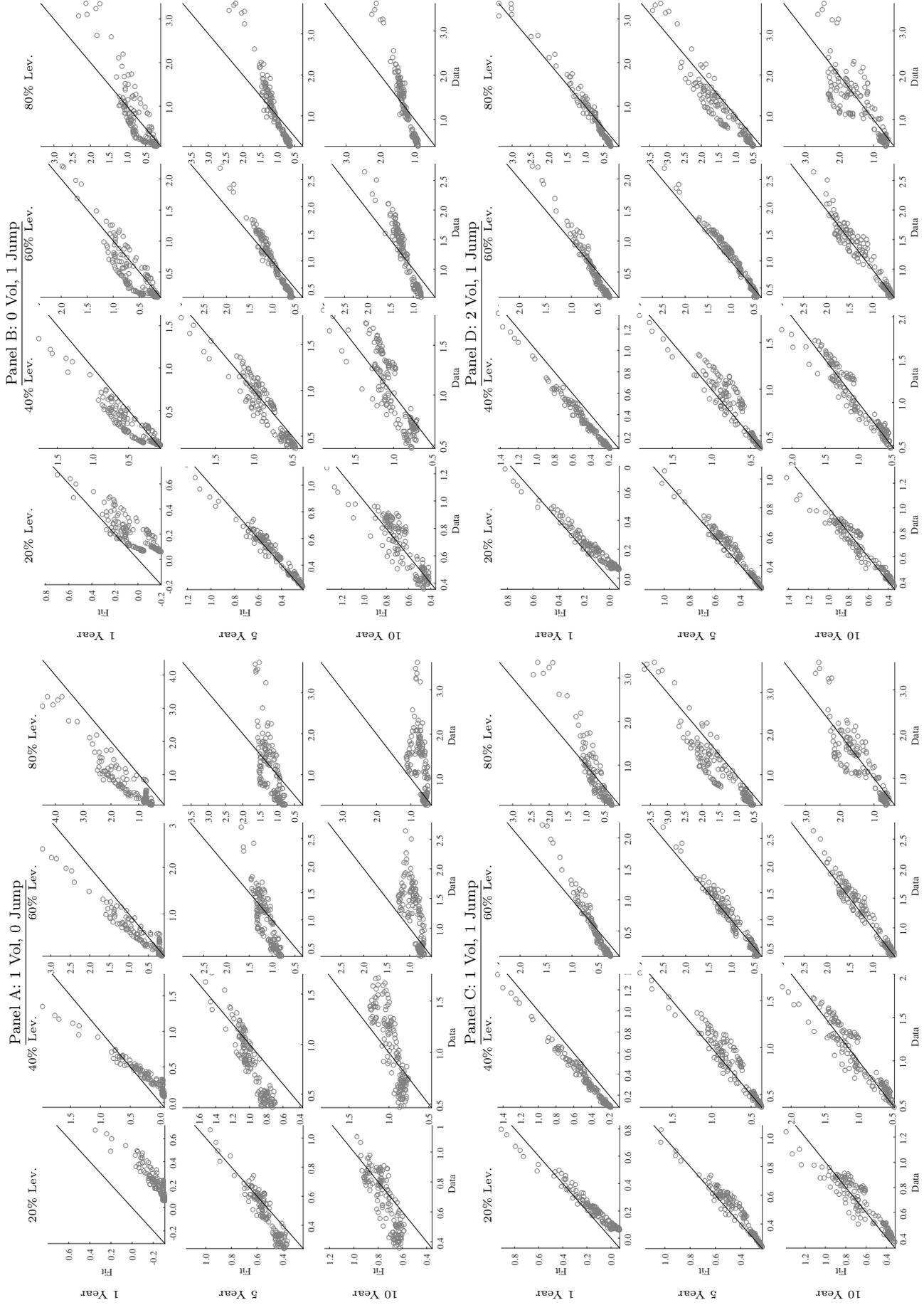
average shape of the CIV surface. Figure 11 plots the average CIV smirk for each maturity. CIV data values are shown in blue and predicted values from each model are in red. Overall, all models successfully generate a steep CIV moneyness slope that is quantitatively consistent with the data. Only the “1 Vol, 0 Jump” specification noticeably underperforms the other versions, and its biggest flaw is underpricing highly levered CDS at long maturities.

Moving beyond unconditional averages, Figures 12 and 13 show the actual versus fitted spreads in each month, observation-by-observation. These figures plainly reveal where each model fails. A pure stochastic volatility specification (Panel A) fails because it cannot generate meaningful time variation in spreads at maturities beyond one year, as seen from their flat pattern in the Figure 12 scatters.<sup>37</sup> This failure is less severe in the jump-only model (Panel B), where the model’s failure is concentrated primarily in the 80% leverage (at-the-money) portfolio. In Panel C, the “1 Vol, 1 Jump” model matches spread behavior across the board with one exception: one-year CDS for high leverage firms. The “2 Vol, 1 Jump” specification (Panel D) remedies this deficiency to provide an accurate fit of all portfolios in every month of our sample.

Figures 12 and 13 emphasize the fits in terms of spreads. We next analyze the models’ ability to match the main dynamic features of the CIV surface documented in Section 4. Each month

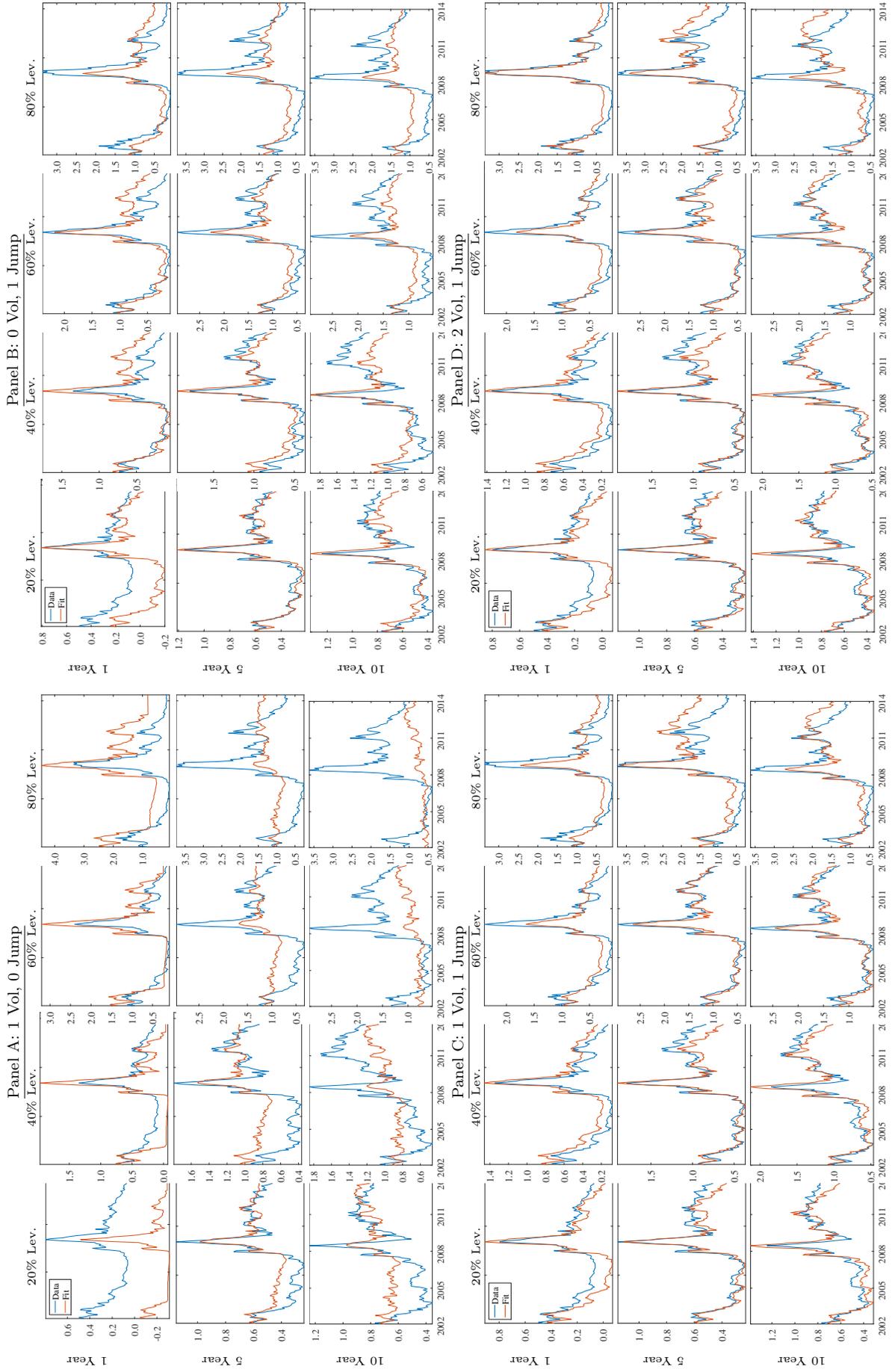
<sup>37</sup>One may fix this by increasing the persistence of volatility (by lowering  $\kappa_{v_1}$ ), but this would in turn force the model to have smaller volatility shocks (lower  $\sigma_{v_1}$ ), which would make the model incapable of matching the level of short-maturity spreads. The tension between persistence and shock variance in our CIR processes arises from the “Feller condition,”  $2\kappa\theta - \sigma^2 > 0$ , which must be satisfied in order to keep the stochastic volatility and jump intensity strictly positive.

Figure 12: STRUCTURAL MODEL FITTED SPREADS



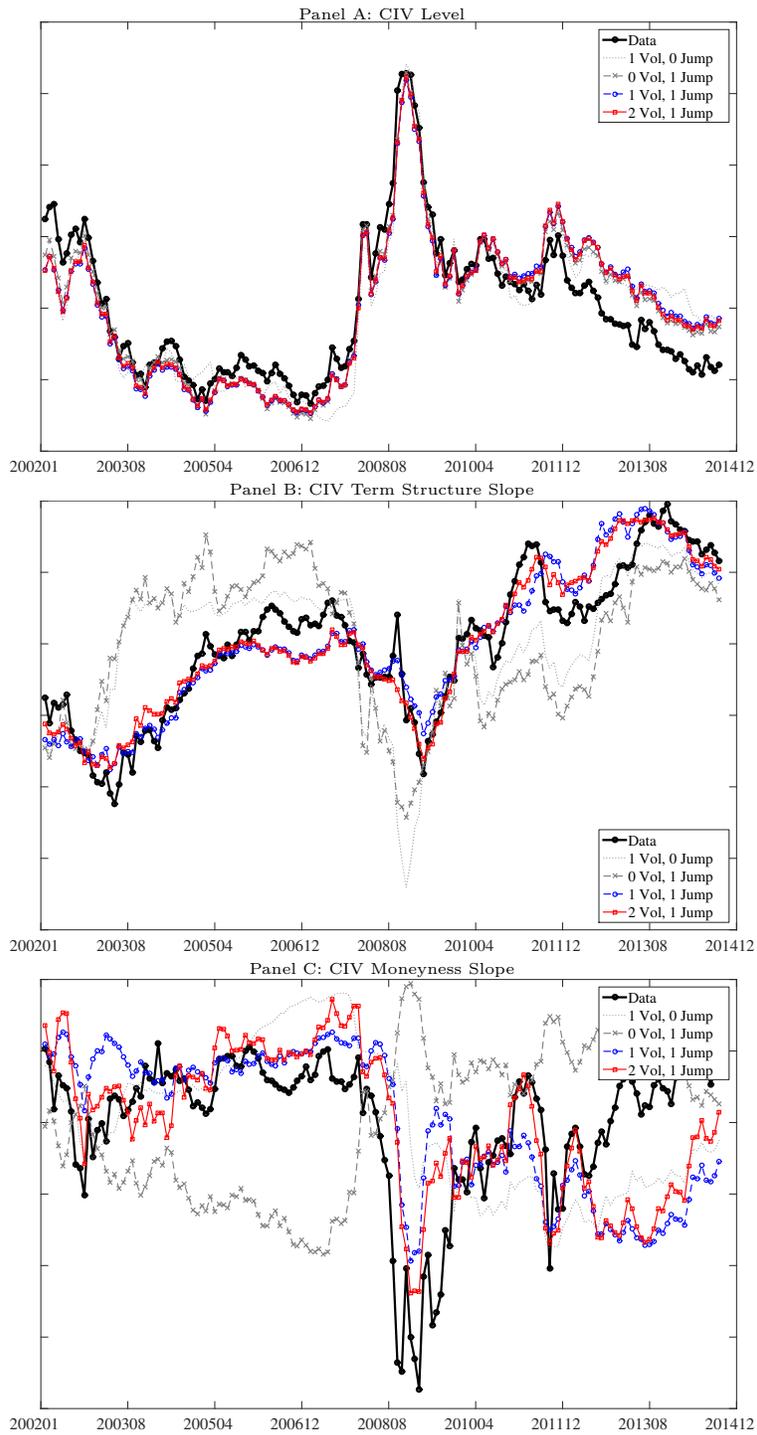
**Note.** Fitted versus actual spreads for 20 constant-leverage and constant-maturity portfolios.

Figure 13: STRUCTURAL MODEL FITTED SPREADS (TIME SERIES)



**Note.** Fitted versus actual spreads for 20 constant-leverage and constant-maturity portfolios.

Figure 14: MODEL-BASED CIV SURFACE DYNAMICS



**Note.** Monthly CIV surface level, moneyness slope, and term structure slope for 20 portfolios in the data and four variations of the structural model.

Table 5: AGGREGATE ASSET PROCESS PARAMETER ESTIMATES

	1 Vol, 0 Jump	0 Vol, 1 Jump	1 Vol, 1 Jump	2 Vol, 1 Jump
$\kappa_{v_1}$	1.920		0.109	0.238
$\theta_{v_1}$	0.030		0.008	0.003
$\sigma_{v_1}$	0.340		0.0001	0.0001
$\rho_{v_1}$	-0.999		-0.952	0.999
$\kappa_z$		0.052	0.564	0.007
$\theta_z$		0.063	0.002	0.041
$\sigma_z$		2e-06	0.039	2e-06
$\mu_{q_m}$		0.852	0.710	0.666
$\sigma_{q_m}$		1.366	2.072	1.999
$a$			0.829	0.612
$\kappa_{v_2}$				0.976
$\theta_{v_2}$				0.005
$\sigma_{v_2}$				0.134
$\rho_{v_2}$				0.999

**Note.** Structural model parameter estimates for the aggregate asset growth process.

we re-calculate the average CIV surface level, moneyness slope, and term structure slope using the fitted values for portfolio CIV based on the estimated model, and plot these alongside the data values in Figure 14. The figures show that all four models do an equally good job of describing the average CIV level (Panel A). However, the models with a single state variable fail to capture the distinct variation in the term structure slope and the moneyness smirk (Panels B and C, respectively). The two-state and three-state models are essentially identical in describing the term structure slope, but the three-state model provides a somewhat better fit of the moneyness smirk.

Comparing Figure 11 versus Figures 12 to 14 shows how unconditional averages can mask poor model performance. It is common practice in the credit pricing literature to estimate and compare models based on their fit of unconditional moments. In our setting, all four models look at least reasonable in terms of their average spreads and CIV. But one-factor models look substantially worse when evaluated on their ability to match the dynamic behavior of spreads—raising a cautionary note for credit analysts who focus on unconditional average fits.

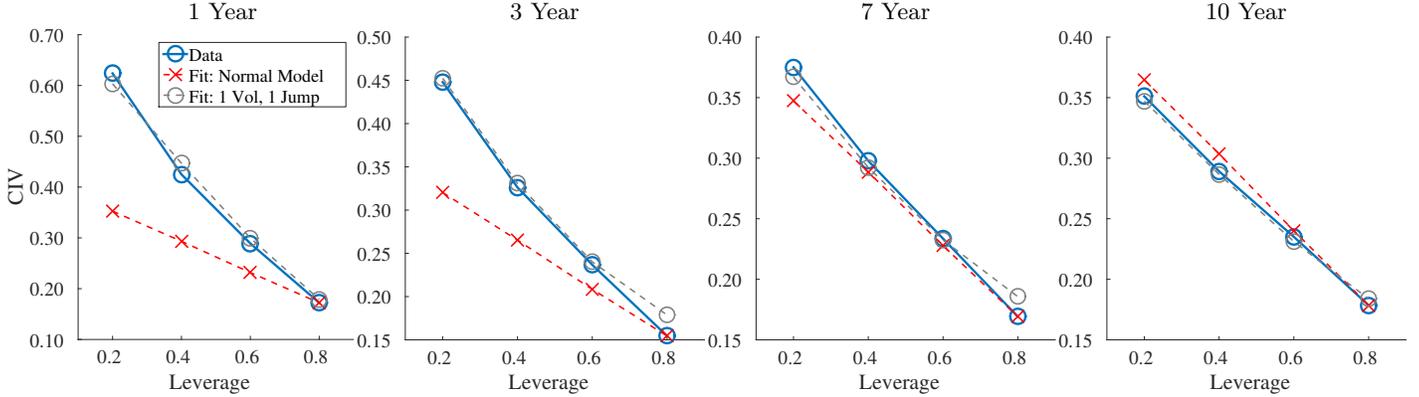
Table 5 reports the fitted risk-neutral parameters of the aggregate asset growth process in each model. We focus our discussion on parameters in the “1 Vol, 1 Jump” specification, which achieves fits nearly as accurate as the three factor model but with less complexity. The stochastic variance process has a long-run mean of  $\theta_{v_1} = .008$ , so on average the diffusive volatility of aggregate asset

growth is about 9% per year. The typical shock to  $v_{1,t}$  has a magnitude of about 1% in annualized volatility terms and is highly persistent (a half-life of 6.3 years). As previous results pointed out, the key model component for matching credit spreads is the jump process. Our estimation looks much like a rare disaster model under the risk-neutral measure. Aggregate jumps arrive on average once every 116 years based on the mean intensity of  $a\theta_{v_1} + \theta_z$  and when they arrive they are potentially cataclysmic. The log jump size has an expected value of  $-71\%$  ( $\mu_{q_m}$  is the magnitude of the downward jump) and a standard deviation of 207%. Why the empirical patterns lead to these estimates can be seen from the heterogeneity-adjusted CIV smirk in Figure 3. Extreme jumps add more mass to the far reaches of the asset growth distribution compared to a normal model. The smirk in Figure 3 is flat for closer-to-the-money firms whose default risk is more closely tied to moderate recessions. CIV smirk bends most steeply upward for very low leverage firms. That is, credit risk is comparatively overpriced in the CDS of firms like Apple and Google, whose historical leverage ratios are below 20% and realistically will only default in catastrophic states of the world. The model reads this CIV smirk from the spreads and interprets it as a (risk-neutral) rare disaster specification.

Our final model-based analysis investigates the extent to which the CIV moneyness smirk is generated by excess kurtosis of asset growth versus being generated by heterogeneity across firms. A purely empirical attempt to disentangle the drivers of the smirk is necessarily confounded by the tendency of firms with different leverage to systematically differ on other dimensions as well, as discussed in Section 4. Working within a structural setting provides an alternative way to understand the nature of the smirk. The model takes a stance on the nature of firm heterogeneity, which in our model enters through heterogeneous risk exposures and idiosyncratic volatilities ( $\beta_i$ ,  $v_i$ , and  $\gamma_i$ ), and takes a stance on the form of non-normalities, entering through stochastic volatility and jump risk.

We work from our estimated “1 Vol, 1 Jump” model which incorporates both underlying firm heterogeneity and excess kurtosis, and study their effects on the CIV smirk by shutting down non-normalities in the model. To remove the effect of non-normalities, we set the variance of the hypothetical normal asset growth distribution equal to observed CIV. With normally distributed

Figure 15: HETEROGENEITY VS. NON-NORMALITY IN MONEYNES SLOPE



**Note.** Actual and model-predicted moneyness smirk for 20 constant-leverage and constant-maturity portfolios.

aggregate and idiosyncratic shocks, the CIV of portfolio  $i$  must obey

$$CIV_{i,t} = \sqrt{(\beta_i^2 + \gamma_i)CIV_{m,t}^2 + v_i}, \quad (7)$$

where  $CIV_{m,t}$  is the normal variance of aggregate asset growth. Under normality, differences in CIV across portfolios must arise solely from heterogeneity in  $\beta_i$ ,  $v_i$ , and  $\gamma_i$ . Thus, we construct a hypothetical “normal” CIV smirk, one that can only be driven by portfolio heterogeneity, by inverting (7) for  $CIV_{m,t}$  from the 80% leverage portfolio (taking direct estimates of  $\beta_i$ ,  $v_i$ , and  $\gamma_i$  as given) and using it to build the hypothetical CIV for the 20%, 40% and 60% leverage portfolios.

The results are shown in Figure 15. The original data is in blue, and the non-normal model fit is in gray. The fitted smirk due to heterogeneity alone—shutting down non-normalities—is in red. For the one-year maturity, heterogeneity explains roughly 40% of the smirk slope, the remainder 60% arising due to non-normality in the asset growth distribution. At longer maturities non-normality plays a smaller role and by ten years heterogeneity can explain essentially the entire smirk, consistent with the patterns in Figure 3.

## 6 Extensions

### 6.1 Bond CIV

This subsection studies CIV calculated from corporate bond spreads rather than from CDS. We use data on option-adjusted spreads (OAS) for US corporate bonds from the Merrill Lynch Corporate Bond Database.<sup>38</sup> To maintain comparability to our earlier results, we restrict our analysis to bonds of companies that also appear in our CDS sample. We use senior unsecured bonds and exclude bonds that are close to being called (where option-adjusted duration is less than 50% of the non-adjusted duration).

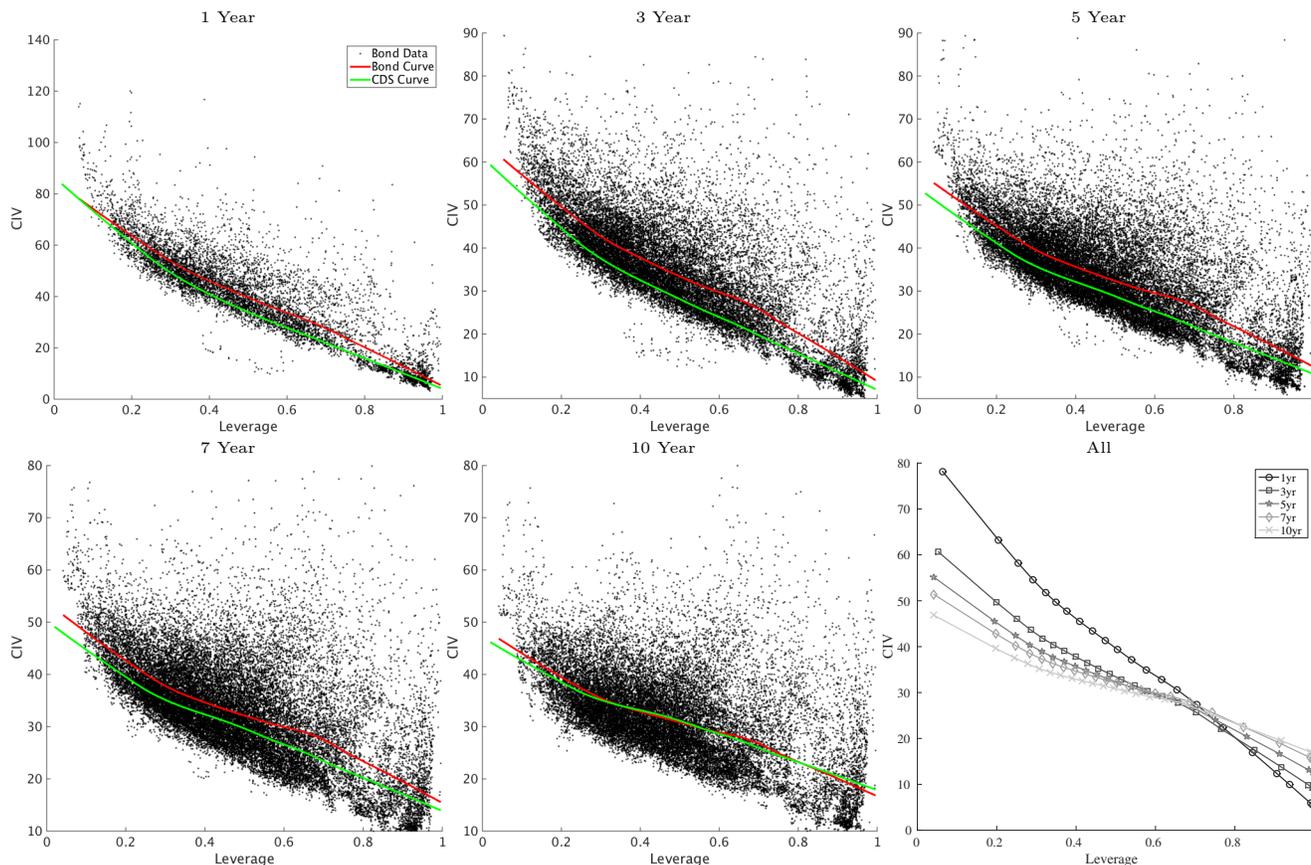
From OAS, we calculate bond-level CIV in the same manner as previous sections. Then, for each firm, we interpolate CIV to match the same maturity grid that we have for CDS (1, 3, 5, 7, and 10 years). If an issuer has bonds with maturities that straddle a grid point then we linearly interpolate CIV. If a firm’s longest bond (call this bond “long”) has maturity no more than 6 months below a grid point, the firm’s CIV at this nearest grid is set to the CIV of “long”, else the CIV at that grid point is designated missing. If a firm’s shortest bond (call this bond “short”) has maturity no more than 6 months beyond a grid point, the firm’s CIV at this nearest grid is set to the CIV of “short”, else the CIV at that gridpoint is designated missing. Our final sample includes bond CIV for a subset of 405 out of 530 firms in our main CDS sample.

Figure 16 plots the moneyness smirk at each maturity for CIV extracted from bond spreads. As in our earlier analysis, we fit a non-parametric curve to the bond CIV scatter plot, which is shown in red. We also overlay on these plots the curve fitted to the CDS data in green. Comparing these two curves shows that the average shape of the CIV surfaces from bonds and CDS are qualitatively identical and are quantitatively very similar, establishing that the credit patterns we document are not unique artifacts of CDS markets.

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<sup>38</sup>This data is used in a number of papers studying bond spreads such as Shaefer and Strebulaev (2008) and Acharya, Amihud, and Bharath (2013). It has a few advantages over other data sets such as TRACE. Most papers that use TRACE drop callable bonds, which tends to remove high yield bonds from the sample as these are typically callable. Also, the Merrill Lynch data is less stale because it includes quotes, while TRACE only has traded prices. Many bond quotes are updated daily though they are not necessarily traded.

Figure 16: BOND CIV SMIRK



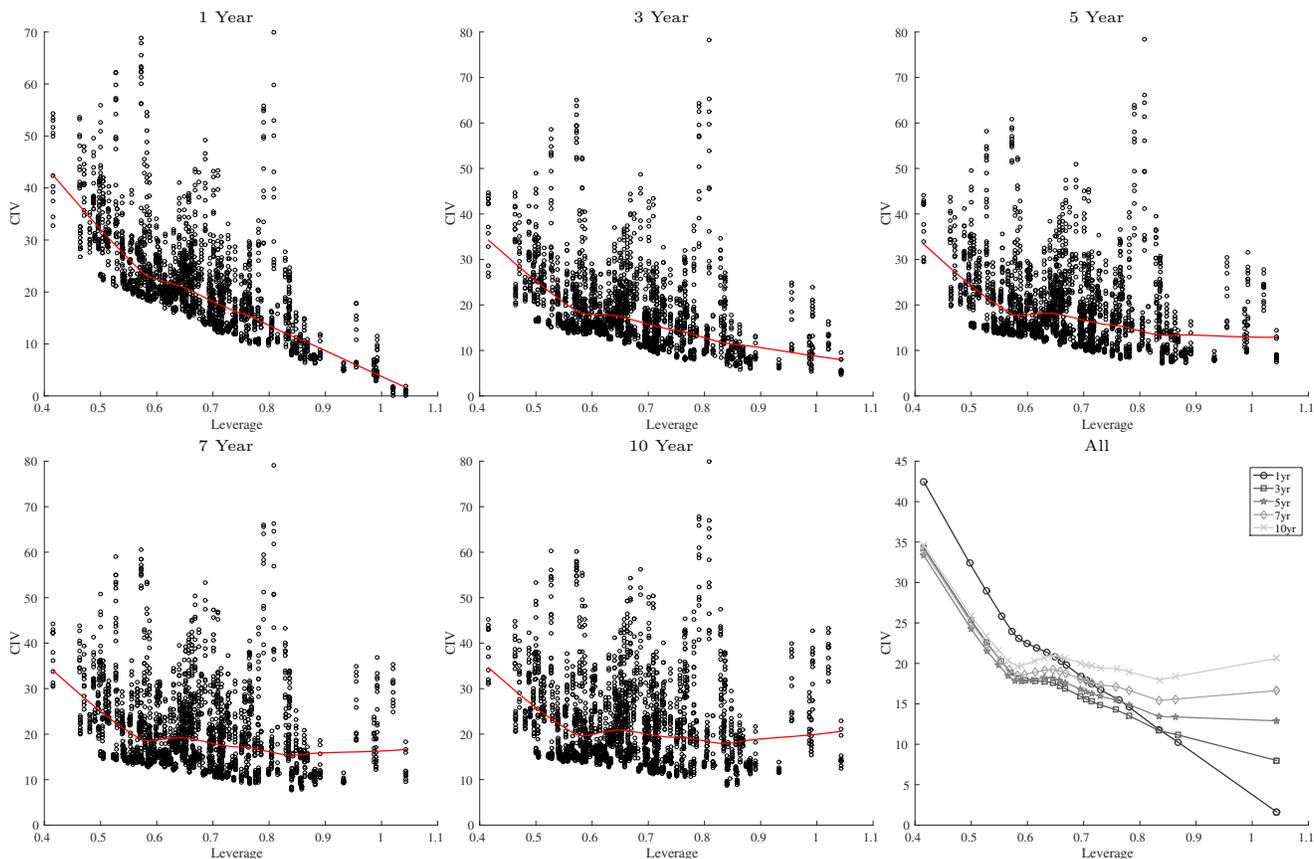
**Note.** Pooled scatter plots of monthly CIV versus firm leverage. In this figure, CIV is inverted from corporate bond spreads rather than CDS spreads. The red line is a fitted non-parametric curve for the bond CIV data. The green curve is the fitted curve based on CDS data taken from Figure 2. The lower right panel overlays the fitted bond CIV curve at all maturities to trace out the bond CIV surface.

## 6.2 Sovereign CIV

We next investigate the role of leverage in the relative pricing of sovereign CDS. Our leverage data are derived from the OECD’s consolidated national balance sheets for general government (OECD data table 710). This includes line items for total government financial assets and liabilities. [OECD \(2015\)](#) states that

[T]he difference between the financial assets and liabilities held by governments (also known as financial net worth or as a broad description of net government debt), gives an extensive measure of the government’s capacity to meet its financial obligation. While financial assets reflect a source of additional funding and income available to government, liabilities reflect the debts accumulated by government. Thus, an increase in the financial net worth signals good financial health. Net worth may be depleted

Figure 17: SOVEREIGN CIV SMIRK



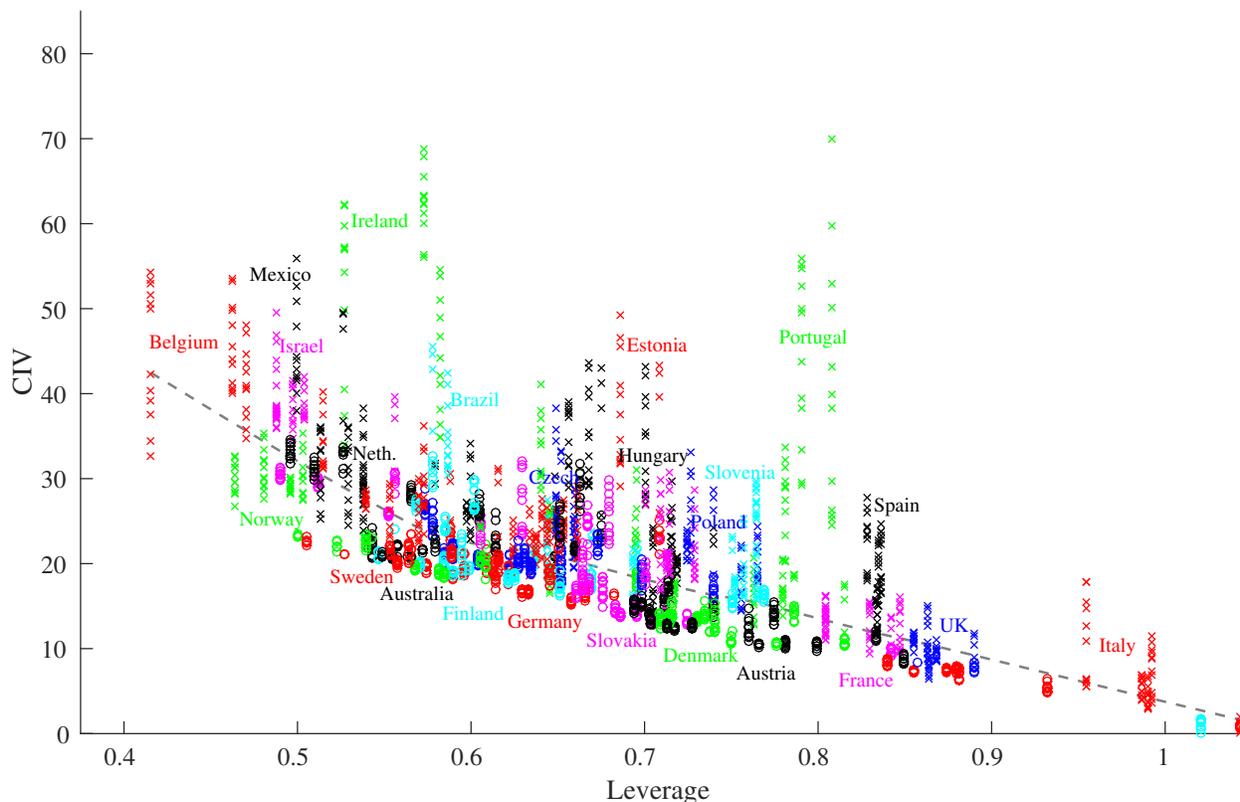
**Note.** Pooled scatter plots of monthly sovereign CIV versus sovereign leverage. The red line is a fitted non-parametric curve. The lower right panel overlays the fitted CIV curve at all maturities to trace out the sovereign CIV surface. Leverage is calculated from OECD consolidated balance sheets for general government. The sample includes Australia, Austria, Belgium, Brazil, Czech Republic, Denmark, Estonia, Finland, France, Germany, Hungary, Ireland, Israel, Italy, Mexico, Netherland, Norway, Poland, Portugal, Slovakia, Slovenia, Spain, Sweden, and the UK, from 2002 to 2012.

by debt accumulation, indicating a worsening of fiscal position and ultimately forcing governments to either cut spending or raise taxes.

We thus define sovereign leverage from OECD data as the ratio of total financial liabilities (net of shares and other equity) to financial assets for the general government sector. Given leverage, we invert CDS spreads for sovereign CIV via the Merton model. Our sample includes 24 countries from 2002 to 2012 for which we have both OECD balance sheet data and sovereign CDS spreads, listed in Figures 17 and 18.<sup>39</sup>

<sup>39</sup>We omit Greece which experienced a sovereign default during our sample and carried a leverage ratio of at least 1.7 since 2002.

Figure 18: SOVEREIGN CIV SMIRK DETAIL



**Note.** Monthly CIV from sovereign CDS with one-year maturity from 2002 to 2012. Circles represent observations through July 2008 and crosses are August 2008 onward. Sovereign leverage is from the OECD. The dashed grey line is a fitted non-parametric curve.

Figure 17 plots the moneyness smirk for sovereign CIV at maturities up to ten years. The lower right panel overlays estimated non-parametric curves for each maturity to trace out the three-dimensional sovereign CIV surface. The general shape of the surface is the same in sovereign and corporate credit markets, with the moneyness slope steepest at one year and gradually flattening out with maturity.

Figure 18 provides more detail on the composition of the data cloud by country and by subsample for one-year CDS. Countries are color-coded and labeled. We distinguish observations before the financial crisis (January 2002 through July 2008, shown with circles) from observations after the start of the crisis (August 2008 through December 2012, shown with crosses). The figure shows that the points lying far above the main smirk curve are all crisis observations.<sup>40</sup> They also are associated with a particular group of distressed sovereigns. The most conspicuous outliers are

<sup>40</sup>The seemingly neglected blue circles on the lower left of Figure 18 are observations from Brazil in 2004 and 2005.

Ireland and Portugal, who were explicitly bailed out by the EU and IMF (other countries in our sample that received EU/IMF bailout funds are Hungary and Spain, and also have large crisis CIV). Another large departure from the curve is Estonia, who suffered a 14% contraction of GDP in 2009 and implemented an “internal devaluation.”

We also find that sovereign CIV possesses a similar factor structure to that for corporate CIV. In our panel of countries, the first PC explains 86% of panel variation in CIV, the second PC explains 10%, and the third explains 2%, consistent with the findings of Longstaff et al. (2011). The similarity in patterns of corporate and sovereign CIV suggests scope for a model of sovereign credit risk, akin to that of corporates in Section 5, in which an individual sovereign’s asset growth is driven by exposure to a global asset growth factor that is subject to heavy-tailed shocks under the risk-neutral pricing measure.

### 6.3 Sectors and Ratings

A common sampling choice in the bond and CDS literature is to remove financial firms. This filter is presumably motivated by financial intermediaries’ unique capital structure and other notions of bank “specialness.” Another common choice is to remove the government-regulated utility sector.<sup>41</sup>

Figure 19 shows a sector breakout of the moneyness smirk using all monthly CIV observations from five-year CDS (using Markit’s ten-sector classification). The figures clearly illustrate different leverage patterns across industries, with healthcare and technology taking the least leverage and financials and utilities being the most levered. All sectors, however, demonstrate the same basic CIV moneyness smirk. There is no individual industry that single-handedly drives the smirk and no industry that deviates from it.<sup>42</sup>

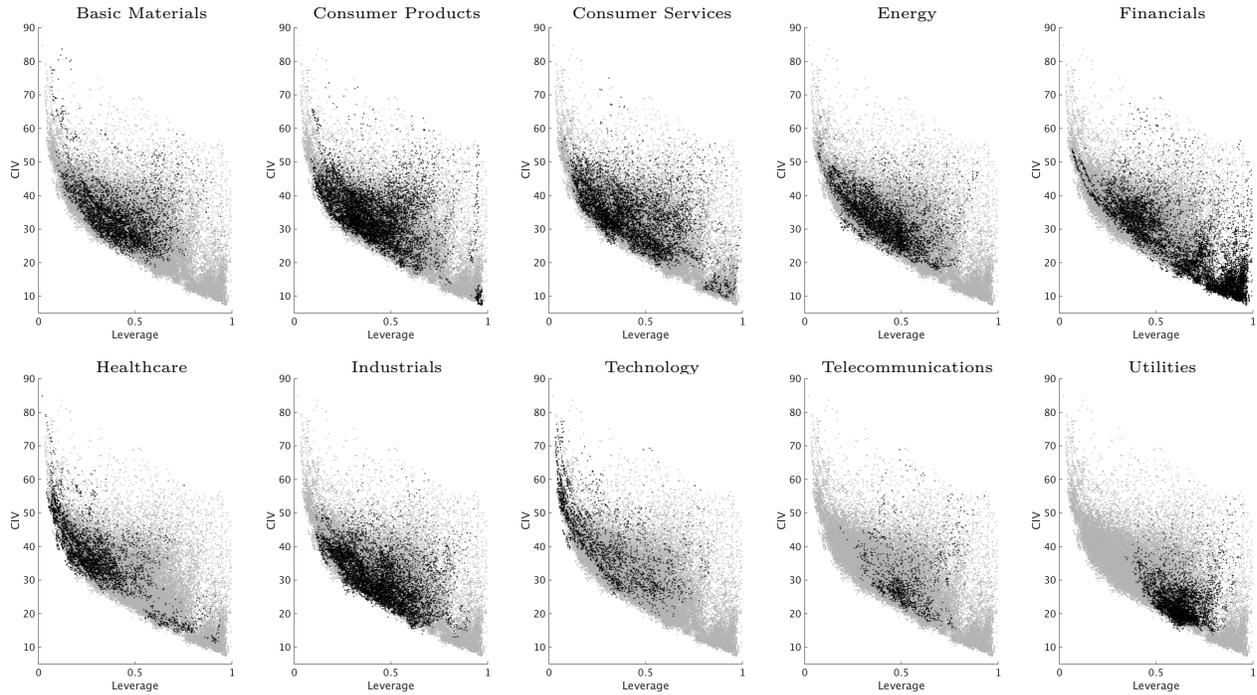
The empirical credit literature also tends to exclude speculative grade bonds. Our main analysis includes BB, which is the highly rated end of the junk spectrum and constitute a relatively large portion (15.9% by observation count) of the available sample, and excludes rating categories B and CCC (accounting for 10% and 6.6% of the available sample, respectively). Figure 20 breaks out the CIV smirk for five-year CDS by rating category. It shows that deviations from the main moneyness

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<sup>41</sup>Schaefer and Strebulaev (2008); Collin-Dufresne, Goldstein and Martin (2001); Bharath and Shumway (2008) exclude financial firms. Chava and Purnanandam (2010); Chen et al. (2015) exclude utilities in addition to financials.

<sup>42</sup>This finding is related to Atkeson, Eisfeldt and Weill (2013), who argue that the credit risk of large financial firms behaves similarly to that of large non-financial firms.

Figure 19: CIV SMIRK BY SECTOR

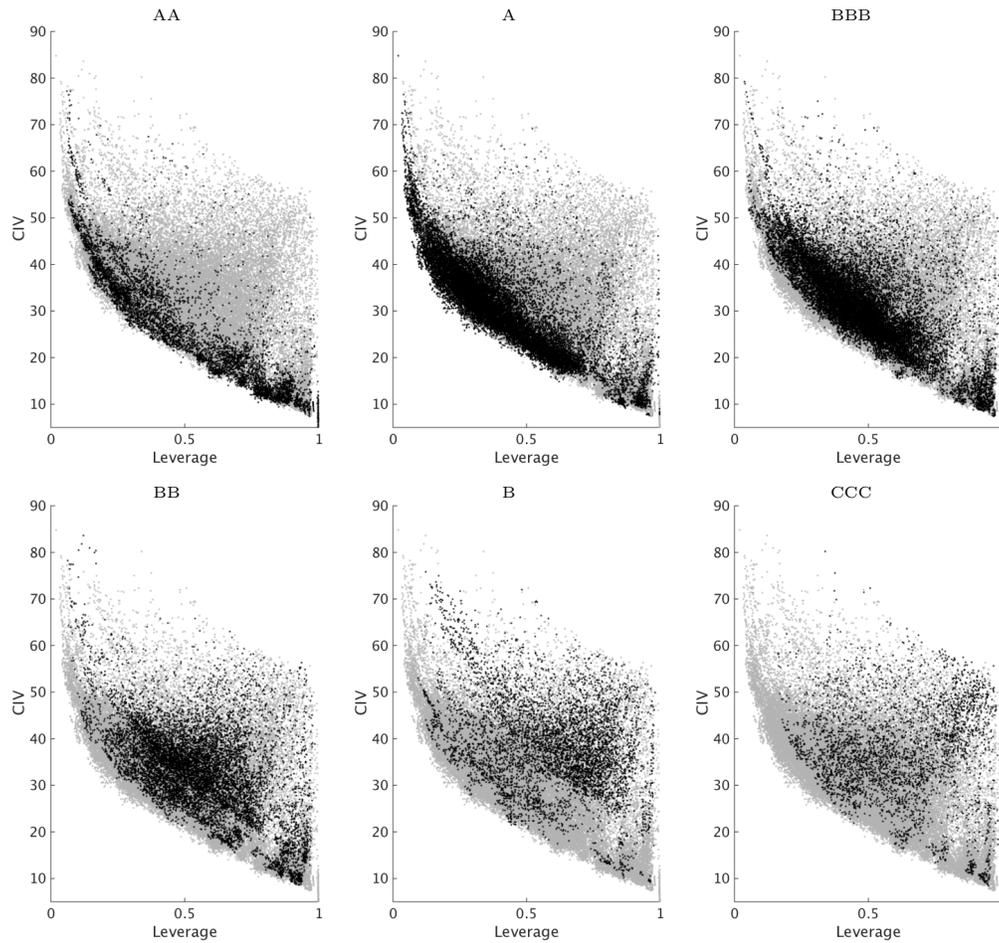


**Note.** CIV from five-year CDS broken out by Markit sector pooling all firm-months. For comparison, the scatter for the full sample combined is shown as gray background in every sub-plot.

smirk are larger the lower rated a firm's credit, lending some credence to the view that junk names demonstrate qualitatively different behaviors than investment grade. While speculative grade firms have unconditionally higher CIV (especially at very high leverage), their dynamic CIV patterns share some similarity to those for investment grade firms. We calculate the time series correlation of monthly average CIV for AA to BB credits with the monthly average for B to CCC, and find a correlation of 70%. This, however, contrasts with the fact that CIV among firms in investment grade rating categories tend to have correlations well over 90% with other investment grade firms. The comparatively low correlation between speculative and investment grade CIV again suggests that distressed credit is subject to forces other than simple default risk.<sup>43</sup>

<sup>43</sup>We also find quantitatively similar results to all portions of our main analysis when we include B and CCC firms in our sample (untabulated). The primary difference in this case is that model fits deteriorate for the 90% leverage portfolio at long maturities (7 and 10 years).

Figure 20: CIV SMIRK BY CREDIT RATING



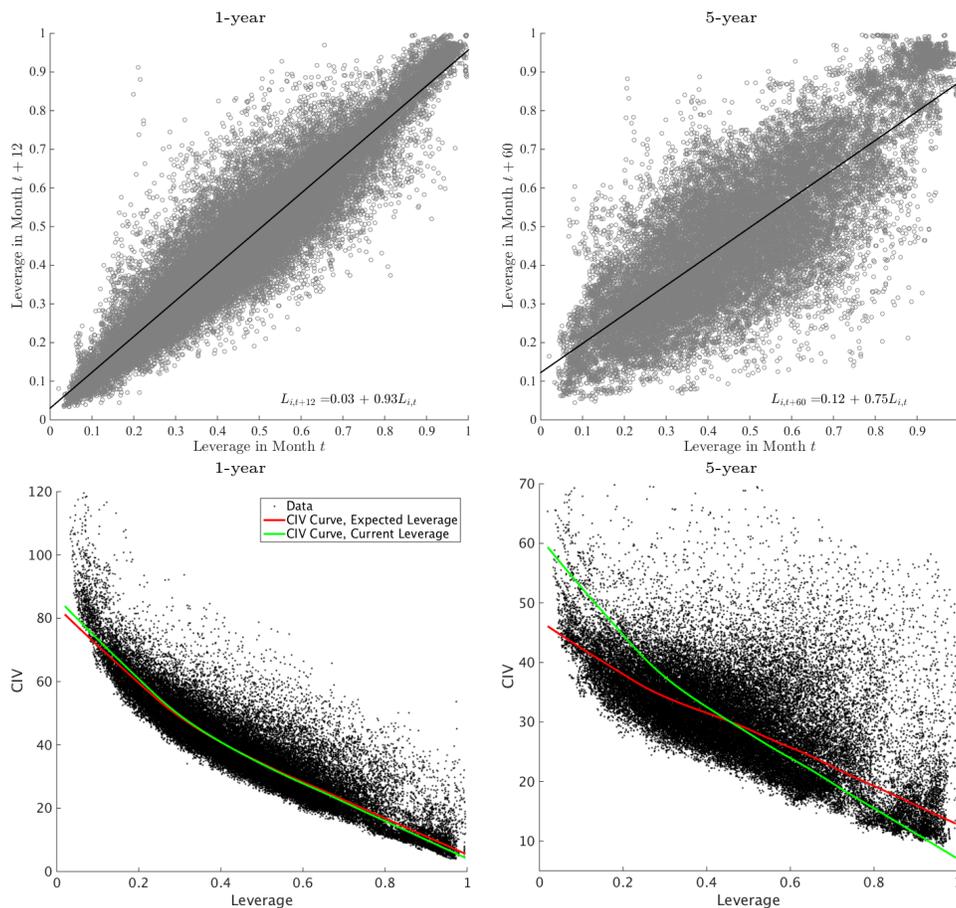
**Note.** CIV from five-year CDS broken out by credit rating pooling all firm-months. For comparison, the scatter for the full sample combined is shown as gray background in every sub-plot.

## 6.4 Mean-reverting Leverage Ratios

A long held tenet of corporate finance is that firms seek to maintain a targeted leverage ratio.<sup>44</sup> A firm experiencing a positive shock to its asset value sees its leverage decline and is likely to issue additional debt in the future to restore the firm's targeted leverage ratio. Similarly, a firm that suffers a loss in asset value will experience high leverage until it has the opportunity to de-lever. In this case, when a firm's leverage is below its targeted level, the *effective* leverage that investors have in mind when valuing CDS is higher than the measured leverage (and vice versa when measured leverage exceeds its targeted value). As a result, calculating CIV from current measured leverage

<sup>44</sup>For empirical evidence, see for example [Hovakimian, Opler and Titman \(2001\)](#). Seminal theoretical work is [Leland \(1994\)](#). [Collin-Dufresne and Goldstein \(2001\)](#) study the theoretical implications for credit spreads.

Figure 21: CIV WITH ANTICIPATED MEAN REVERSION IN LEVERAGE



**Note.** The top two panels plot realized future leverage against current leverage for all firm-months in our sample along with the pooled OLS regression fit. The bottom two panels plot expectations-adjusted CIV (that is, recomputed using expected future leverage based on the regressions in the top panel) against current leverage, along with a fitted non-parametric curve in red. We overlay the corresponding raw CIV curve from Figure 2 in green.

may erroneously produce a moneyness smirk.

We quantitatively investigate this possibility in our data. Let  $L_{i,t}$  denote measured leverage for firm  $i$  in month  $t$ . The top left panel of Figure 21 compares current leverage to realized leverage twelve months into the future. The fitted line in the figure is from a pooled OLS regression of  $L_{t+12}$  on current leverage  $L_{i,t}$ . The result shows, unsurprisingly, that leverage is very sticky at the one year horizon, with an intercept estimate of 0.03 and a slope of 0.93. But these are highly statistically different from 0 and 1, respectively, demonstrating significant mean reversion in leverage ratios.

To understand the effect that expected leverage reversion has on our implied volatility esti-

mates, we recompute one-year CIV using expected future leverage over the one-year life of the CDS contract.<sup>45</sup> The lower left panel of Figure 21 plots this expectations-adjusted CIV against current leverage and fits a non-parametric curve to the data points (shown in red). We overlay on this the corresponding curve using raw CIV from Figure 2 (shown in green). Evidently, expected leverage reversion over one year bears little impact on the shape of the one-year CIV smirk, as the raw and expectations-adjusted smirk almost exactly coincide. The right hand panel repeats these analyses using realized leverage five years into the future. Here, the role of mean-reversion appears more important—the magnitude of the five-year smirk slope is reduced by roughly one-third—but otherwise leaves a steep five-year CIV smirk intact.

## 7 Conclusion

We present the credit implied volatility surface as an organizing framework for empirical analysis of credit spreads. We show that most of the variation in the relative cost of a credit instrument lines up with moneyness of the contract, summarized by the underlying firm’s leverage ratio, and the contract’s maturity. We document a steep CIV moneyness slope that implies large deviations from normality in the risk-neutral distribution of aggregate asset growth. Dynamics of the CIV surface can be summarized with a few common factors, interpretable as CIV level, term structure slope, and moneyness slope, that provide a compact and complete description of time-variation in the entire panel of firm-level credit spreads. We also show that the cross section of CDS risk premia align with moneyness and maturity of the CDS and is fully explained by exposure to the CIV level factor. Finally, we show that a parsimonious structural model with stochastic volatility and jumps provides an accurate description of CDS spreads for firms across the credit spectrum, at short and long maturities, and at all points throughout the credit cycle. Our estimation suggests that the risk-neutral distribution of aggregate asset growth can be effectively described as a rare disaster model.

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<sup>45</sup>We set expected leverage equal to the midpoint of current leverage and predicted one-year-ahead leverage estimated from the regression line in Figure 21.

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# Appendix

## A Measuring Realized Growth in Asset Values

In this appendix we describe our calculation for growth rates in firm-level asset values at the daily frequency. We use this in Section 4 to construct measures of firm-level asset riskiness for the regressions in Table 2 and for the construction of heterogeneity-adjusted CIV in Figure 3. We also use this data in Section 5 to estimate portfolio asset betas and idiosyncratic asset growth volatility.

The return on the (market value) of firm’s assets is equal to the weighted average return on its debt and equity, where the weights are defined by the firm’s capital structure. The difficulty in observing asset values stems from the fact that most companies have non-public debt or debt-like obligations (e.g. bank loans, supplier credit, and other liabilities) therefore one cannot simply look at the value of the bonds a company has outstanding. Even for firms that have most of their debt in bonds, it is unlikely that all those bonds will be liquid, introducing problems of synchronicity.

To circumvent this problem, we approximate asset returns by combining information from CDS and equity prices. The key step in our construction approximates the return on debt with a portfolio consisting of a short CDS and a long risk-free bond. Because CDS are swap contracts, the daily return on a CDS is defined as

$$r_{CDS,i,t} = \frac{S_{i,t}}{250} - \text{Dur}_{CDS,i,t} \Delta S_{it}.$$

The first term captures the “carry” component of the return, or fraction of the annual swap payment that the CDS seller accrues each day. The second term captures the capitalized gain/loss from a change in the CDS spread, with  $\text{Dur}_{CDS,i,t}$  adjusting for the risky duration of the CDS. Our CDS return calculation uses a firm’s five-year CDS spread, and we calculate the CDS risky duration according to [Palhares \(2013\)](#). The CDS return is then combined with the risk-free return, calculated as the return to an interpolated four-year duration Treasury bond and denoted  $r_{f,t}$ . Next, to approximate a firm’s return on debt, the CDS and risk-free returns within the portfolio are combined so that the portfolio’s duration matches the duration of the firm’s debt,  $\text{Dur}_{D,i,t}$ , which is calculated as the capital-weighted duration across all bonds issued by a firm. In particular, we calculate the return on debt as

$$r_{D,i,t} = \frac{\text{Dur}_{D,i,t}}{\text{Dur}_{CDS,i,t}} r_{CDS,i,t} + \frac{\text{Dur}_{D,i,t}}{\text{Dur}_{f,t}} r_{f,t}.$$

This in turn allows us to approximate the realized physical return on total assets as

$$r_{A,i,t} = \frac{D_{i,t}}{A_{i,t}} r_{D,i,t} + \frac{E_{i,t}}{A_{i,t}} r_{E,i,t},$$

and we aggregate firm-level asset returns into constant-leverage portfolios to corresponding to those used in our model estimation. We calculate measures of asset riskiness from these daily asset returns within each month.

## B Pricing Derivation for Structural Model

In this appendix we present and derive the structural model we use to fit the panel of CDS spreads. We derive a closed-form solution for the characteristic function of the probability of finishing in-the-money, and use the Fourier transform to recover this probability, following [Duffie, Pan and Singleton \(2000\)](#) and [Carr and Madan \(1999\)](#).

Denote the log asset process as  $Y_{i,t} = \ln A_{i,t}$ , where the dynamics of  $A_{i,t}$  are defined in Section 5. Define the characteristic function  $\varphi(u, t, T)$  as<sup>46</sup>

$$\varphi(u, t, T) \equiv \mathbb{E}_t^{\mathbb{Q}} \left[ e^{iuY_T} \right].$$

By Ito's lemma, the dynamics of  $Y_{i,t} = \ln A_{i,t}$  are

$$\begin{aligned} dY &= \left( r - \frac{1}{2}\beta_i^2 v_{1,t} - \frac{1}{2}\beta_i^2 v_{2,t} - \frac{1}{2}v_{i,t} - (1 + \beta_i) \lambda_{m,t} \xi_m \right) dt \\ &+ \beta_i \left( \sqrt{v_{1,t}} dW_t^1 + \sqrt{v_{2,t}} dW_t^2 + (e^{-q_m} - 1) dJ(\lambda_{m,t}) \right) \\ &+ \sqrt{v_{i,t}} dW_t^i + (e^{-q_m} - 1) dJ(\lambda_{m,t}). \end{aligned}$$

An application of the Feynman-Kac theorem to the above characteristic function gives the following expected PDE:

$$\begin{aligned} \mathbb{E}[d\varphi(u, t, T, Y)] &= \frac{\partial \varphi}{\partial t} + \frac{\partial \varphi}{\partial Y} \left( r - \frac{1}{2}\beta_i^2 v_{1,t} - \frac{1}{2}\beta_i^2 v_{2,t} - \frac{1}{2}v_{i,t} - \lambda_{i,t} \xi_i - \beta_i \lambda_{m,t} \xi_m \right) dt \\ &+ \frac{1}{2} \frac{\partial^2 \varphi}{\partial Y^2} \left( \beta_i^2 (v_{1,t} + v_{2,t}) + v_{i,t} \right) dt \\ &+ \frac{\partial \varphi}{\partial v_1} k_{v_1} (\theta_{v_1} - v_{1,t}) dt + \frac{1}{2} \frac{\partial^2 \varphi}{\partial v_1^2} \sigma_{v_1}^2 v_{1,t} dt + \sigma_{v_1} v_1 \beta_i \rho_{Y v_1} \frac{\partial \varphi^2}{\partial Y \partial v_1} \\ &+ \frac{\partial \varphi}{\partial v_2} k_{v_2} (\theta_{v_2} - v_{2,t}) dt + \frac{1}{2} \frac{\partial^2 \varphi}{\partial v_2^2} \sigma_{v_2}^2 v_{2,t} dt + \sigma_{v_2} v_2 \beta_i \rho_{Y v_2} \frac{\partial \varphi^2}{\partial Y \partial v_2} \\ &+ \frac{\partial \varphi}{\partial z} k_z (\theta_z - z_t) dt + \frac{1}{2} \frac{\partial^2 \varphi}{\partial z^2} \sigma_z^2 z_t dt + \sigma_z z \rho_{Y z} \frac{\partial \varphi^2}{\partial Y \partial z} \\ &- (1 + \beta_i) \lambda_{m,t} \mathbb{E}[\varphi(y) - \varphi(y + q_m)]. \end{aligned}$$

Conjecture the following solution that is exponential in the state variables

$$\varphi(u, t, T) = e^{A(T-t) + B(T-t)v_{1,t} + C(T-t)v_{2,t} + D(T-t)z_t + iuY_t}$$

where

$$\begin{aligned} \frac{\partial \varphi}{\partial t} &= \left( -\dot{A} - \dot{B}v_1 - \dot{C}v_2 - \dot{D}z \right) \varphi \quad , \quad \frac{\partial \varphi}{\partial Y} = iu\varphi \quad , \quad \frac{\partial \varphi^2}{\partial Y^2} = -u^2\varphi \\ \frac{\partial \varphi}{\partial v_1} &= B\varphi \quad , \quad \frac{\partial \varphi^2}{\partial v_1^2} = B^2\varphi \quad , \quad \frac{\partial \varphi^2}{\partial Y \partial v_1} = iuB \\ \frac{\partial \varphi}{\partial v_2} &= C\varphi \quad , \quad \frac{\partial \varphi^2}{\partial v_2^2} = C^2\varphi \quad , \quad \frac{\partial \varphi^2}{\partial Y \partial v_2} = iuC \\ \frac{\partial \varphi}{\partial z} &= D\varphi \quad , \quad \frac{\partial \varphi^2}{\partial z^2} = D^2\varphi \quad , \quad \frac{\partial \varphi^2}{\partial Y \partial z} = iuD. \end{aligned}$$

Substituting the above partial derivatives and applying the separation of variables method, we get

<sup>46</sup>For clarity of exposition, we drop the dependence of the characteristic function on state variables.

the following system of ODE's:

$$\begin{aligned}
\dot{A} &= iur - \frac{1}{2}v_i (iu + u^2) + Bk_{v_1}\theta_{v_1} + Ck_{v_2}\theta_{v_2} + Dk_z\theta_z \\
\dot{B} &= -\frac{1}{2}(\beta_i^2 + \gamma_i)(iu + u^2) - (1 + \beta_i)a(iu\xi_m + \psi_m(u)) - B(k_{v_1} - iu\sigma_{v_1}\beta_i\rho_{Y_{v_1}}) + \frac{1}{2}B^2\sigma_{v_1}^2 \\
\dot{C} &= -\frac{1}{2}(\beta_i^2 + \gamma_i)(iu + u^2) - (1 + \beta_i)a(iu\xi_m + \psi_m(u)) - C(k_{v_2} - iu\sigma_{v_2}\beta_i\rho_{Y_{v_2}}) + \frac{1}{2}C^2\sigma_{v_2}^2 \\
\dot{D} &= -(1 + \beta_i)(iu\xi_m + \psi_m(u)) - D(k_z - iu\sigma_z\beta_i\rho_{Y_z}) + \frac{1}{2}D^2\sigma_z^2
\end{aligned}$$

where  $\psi_m(u) = 1 - e^{-iu\mu_{qm} - .5u^2\sigma_{qm}^2}$  is the characteristic exponent. Solutions to the ODEs are

$$\begin{aligned}
B(\tau) &= \frac{c_{v_1} - d_{v_1}}{\sigma_{v_1}^2} \frac{e^{-d_{v_1}\tau} - 1}{fe^{-d_{v_1}\tau} - 1} \\
C(\tau) &= \frac{c_{v_2} - d_{v_2}}{\sigma_{v_2}^2} \frac{e^{-d_{v_2}\tau} - 1}{fe^{-d_{v_2}\tau} - 1} \\
D(\tau) &= \frac{c_z - d_z}{\sigma_z^2} \frac{e^{-d_z\tau} - 1}{fe^{-d_z\tau} - 1} \\
A(\tau) &= iur - \frac{1}{2}v_i (iu + u^2) \\
&\quad + \frac{k_{v_1}\theta_{v_1}}{\sigma_{v_1}^2} \left[ (c_{v_1} - d_{v_1})\tau - 2 \ln \left\{ \frac{\psi_{v_1}(\tau) - 1}{\psi_{v_1}(0) - 1} \right\} \right] \\
&\quad + \frac{k_{v_2}\theta_{v_2}}{\sigma_{v_2}^2} \left[ (c_{v_2} - d_{v_2})\tau - 2 \ln \left\{ \frac{\psi_{v_2}(\tau) - 1}{\psi_{v_2}(0) - 1} \right\} \right] \\
&\quad + \frac{k_z\theta_z}{\sigma_z^2} \left[ (c_z - d_z)\tau - 2 \ln \left\{ \frac{\psi_z(\tau) - 1}{\psi_z(0) - 1} \right\} \right]
\end{aligned}$$

where  $\tau = T - t$ , and

$$\begin{aligned}
c_{v_1} &= k_{v_1} - iu\sigma_{v_1}\beta_i\rho_{Y_{v_1}} \\
d_{v_1} &= \sqrt{c_{v_1}^2 + \sigma_{v_1}^2 [(\beta_i^2 + \gamma_i)(iu + u^2) + 2a(1 + \beta)(\xi_m iu + \psi_m(u))]} \\
c_{v_2} &= k_{v_2} - iu\sigma_{v_2}\beta_i\rho_{Y_{v_2}} \\
d_{v_2} &= \sqrt{c_{v_2}^2 + \sigma_{v_2}^2 [(\beta_i^2 + \gamma_i)(iu + u^2) + 2a(1 + \beta)(\xi_m iu + \psi_m(u))]} \\
c_z &= k_z \\
d_z &= \sqrt{c_z^2 + 2(1 + \beta_i)\sigma_z^2(iu\xi_m + \psi_m(u))} \\
\psi_I(\tau) &= f_I e^{-d_I\tau} && \text{for } I = v_1, v_2, z \\
f_I &= (c_I - d_I)/(c_I + d_I) && \text{for } I = v_1, v_2, z
\end{aligned}$$

Finally, we evaluate put option prices following the numerical procedure suggested by Carr and Madan (1999).

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